Rydberg atoms: excitation, interactions, trapping

Mark Saffman

I: Coherent excitation of Rydberg states

II: Rydberg atom interactions

III: Coherence properties of ground and Rydberg atom traps
1: Coherent excitation of Rydberg states

- overview

- calculation of rates

- 1 photon methods

- 2 photon methods, Doppler, AC Stark shifts

- 3 photon methods

- experiments, coherent oscillations
Overview

- Experiments that use Rydberg atoms require excitation of Rydberg states

- 1, 2 and 3 photon techniques can be used to access low L Rydberg states (high L (circular) states involve special techniques)

- For applications involving Quantum information it is generally necessary for the excitation to be fast, coherent and state selective

- With modern laser systems coherent excitation is relatively straightforward
Few photon excitation

Alkali atom

ground state

ns_{1/2} np_{1/2} np_{3/2}

ns_{1/2} nd_{3/2} nd_{5/2}

ns_{1/2} nf_{5/2} nf_{7/2}

Alkali atom

ground state

ns_{1/2} np_{1/2} np_{3/2}

ns_{1/2} nd_{3/2} nd_{5/2}

ns_{1/2} np_{1/2} np_{3/2}

Alkali atom

ground state

ns_{1/2} np_{1/2} np_{3/2}
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One photon excitation rate

\[ I(r) = \frac{\varepsilon_0 c}{2} |\mathcal{E}(r)|^2 \]

Rabi frequency
\[ \Omega(r) = \frac{d\mathcal{E}(r)}{\hbar} \]

Transition matrix element
\[ d = e \langle r | r_q | g \rangle \]

\[ r_0 = z, \quad r_+ = -\frac{1}{\sqrt{2}}(x + iy), \quad r_- = \frac{1}{\sqrt{2}}(x - iy) \]

Wigner-Eckart theorem
\[ \langle \beta j' m' | r_q | \alpha j m \rangle = \frac{\langle \beta j' | r | \alpha j \rangle}{\sqrt{2j'+1}} C_{j m 1 q}^{j' m'} \]

Alkali atom ground state

Additional complication mixed representations hyperfine – fine structure states
Radial matrix element

Apart the many angular factors we need to calculate radial integrals.
Scaling of radial integrals

Estimate integral in the limit $n \gg 1$:

Integrand vanishes away from the origin.

Assume Rydberg state $ns$, $|R_{ns}(0)|^2 \sim 1/n^3$

Scaling is valid for any low angular momentum Rydberg state. This says optical power $P \sim n^3$ at constant $\Omega$

Calculation methods:
- Coulomb wave functions (q.d. theory, Seaton)
- Model potentials

Radial integral numerics

\[ R_{n' L', n L} = \int_0^\infty dr \ r^3 R_{n' L'} R_{n L} \]

Cs wavefunctions

Integrand is localized near the core

Alkali atom ground state

\( np_{1/2} \)

\( np_{3/2} \)

\( ns_{1/2} \)

\( 50 a_0 \)

\( 5000 a_0 \)

\( r^2 R_{6s} \)

\( r^2 R_{50p} \)
Radial integral numerics

Cs 6s – np^{1/2}

\[ R_{n'L',nL} = \int_0^\infty dr \, r^3 R_{n'L'} R_{nL} \]

- Asymptotic scaling is very good for n>50
- WKB approximation about 2x too small.
- WKB works much better for n,n’ both large (lecture 2)
1: Coherent excitation of Rydberg states

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Single photon excitation

For high lying Rydberg levels wavelength is 297 nm for Rb, 319 nm for Cs.

Not commonly done.

Also strong Doppler sensitivity, we will come back to this.
Optical spectroscopy of rubidium Rydberg atoms with a 297 nm frequency-doubled dye laser

P. Thoumany, T. Hänsch, G. Stania, L. Urbonas, and Th. Becker*
1: Coherent excitation of Rydberg states

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Two photon excitation

With two photon scheme can use longer wavelengths.

Less Doppler if we use counterpropagation.

Drawback spontaneous emission from intermediate level.

For large detuning from $|p>$ the two-photon Rabi frequency is

$$\Omega = \frac{\Omega_1 \Omega_2}{2 \Delta_p}$$

Spontaneous emission in $\pi$ pulse

$$P_{se} = \frac{\pi \gamma_p}{4 |\Delta_p|} (q + 1/q), \quad q = |\Omega_2/\Omega_1|$$

Putting $q=1$

$$\Omega = \frac{P_{se} |\Omega_2|^2}{\pi \gamma_p}$$
Two photon excitation

We found

\[ \Omega = \frac{P_{se} |\Omega_2|^2}{\pi \gamma_p} \]

Thus

\[ \Omega \sim \frac{P_{se} P_2}{\gamma_p} \]

If we have enough optical power we can have fast Rabi frequency with small spontaneous emission.

Common wavelengths:

Rb: \( 5p_{3/2} \): 780 nm, 480 nm
\( 6p_{3/2} \): 420 nm, 1015 nm

Cs: \( 6p_{3/2} \): 852 nm, 510 nm
\( 7p_{3/2} \): 459 nm, 1038 nm

Second resonance levels have smaller \( \gamma_p \).
Two photon excitation – design example for Rb

Figure 8: Rabi frequency for excitation of the $70d_{5/2}$ level for $P_{se} = 10^{-2}, 10^{-3}, 10^{-4}$. The 780 nm, 480 nm approach via $5p_{3/2}$ is shown on the left and the 420 nm, 1010 nm approach via $6p_{3/2}$ is shown on the right. The power requirements are calculated for our current experimental beam size of $w = 4 \mu m$.

M. Saffman 2006

Second resonance excitation of Rb has been implemented here in Pisa.
Two photon excitation - headaches

Doppler broadening:

Copropagation: \[ \delta \omega = \mathbf{v} \cdot (\mathbf{k}_1 + \mathbf{k}_2) \]

Take \( v = 10 \text{ cm/s}, \lambda_1 = 780 \text{ nm}, \lambda_2 = 480 \text{ nm} \)

\[ \delta \omega / 2\pi = 0.34 \text{ MHz} \]
Doppler broadening example

Doppler broadening:

Copropagation:

\[ \delta \omega = v \cdot (k_1 + k_2) \]

Take \( v=10 \text{ cm/s}, \lambda_1=780 \text{ nm}, \lambda_2=480 \text{ nm} \)

\[ \frac{\delta \omega}{2\pi} = 0.34 \text{ MHz} \]

Counterpropagation:

\[ \delta \omega = v \cdot (k_1 - k_2) \]

\[ \frac{\delta \omega}{2\pi} = 0.08 \text{ MHz} \]
Doppler broadening – time domain

Rb 780+480 nm excitation, $\Omega/2\pi = 1$ MHz
Integrate under Doppler curve.

150 $\mu$K

T = 10 mK

Copropagating/counterpropagating

Upper gray line is spontaneous emission limit with Rydberg lifetime 300 $\mu$s, n~ 90
AC Stark shifts

The two-photon transition is AC Stark shifted by the excitation beams. This gives sensitivity to intensity noise on lasers, atomic position under envelope of beam intensity.

![Diagram of energy levels and transitions demonstrating AC Stark shifts](image)

- **Resonant** transitions at 480 nm and 780 nm
- **Non-resonant** transitions
- **Signal counts** as a function of Red AOM double-pass frequency
  - Low power: FWHM = 2.2 MHz
  - High power: FWHM = 4.5 MHz
AC Stark shifts

The two-photon transition is AC Stark shifted by the excitation beams. This gives sensitivity to intensity noise on lasers, atomic position under envelope of beam intensity.

Main contribution is 1st beam on ground state and 2nd beam on Rydberg state, i.e. near resonant interactions.

\[
\Delta_{ac} = \frac{|\Omega_2|^2 - |\Omega_1|^2}{4\Delta_p}
\]

Choose \(\Omega_1 = \Omega_2\) to cancel.
Doppler and AC Stark cancellation

The Doppler shifts depend on velocity. The AC Stark shifts depend on detuning from $|p>$, which also depends on velocity.

It is possible to make these two effects cancel each other.

Interesting, but not that useful for coherent experiments since it requires tuning close to intermediate level.

\[ \frac{\Omega_1^2}{\Omega_1^2 + 2\delta^2} = \frac{\omega_0 - \omega'_0}{\omega_0} \]
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Three photon excitation

Gives access to np and nf, final states

Can choose levels so all wavelengths can be derived from IR laser diodes.

Can choose directions of $k_1$, $k_2$, $k_3$ to cancel Doppler broadening.

Doppler- and recoil-free laser excitation of Rydberg states via three-photon transitions


Coplanar solution

$$k_1 + k_2 + k_3 = 0.$$
Doppler broadening comparison

Rb, $\Omega/2\pi=0.5$ MHz

Three photon scheme gives substantial Rydberg excitation even at room temperature
3 photon excitation

Pillet group

Diode lasers or Ti:Sa

About 10 MHz linewidth, may also be broadened by interactions

Kinetic Monte Carlo modeling of dipole blockade in Rydberg excitation experiment

Amodsen Chotta¹, Matthieu Viteau, Thibault Vogt, Daniel Comparat and Pierre Pillet

3 photon excitation with 2 photon degeneracy

Spectroscopy of rubidium Rydberg states with three diode lasers

1256 nm
5 D $^5/2$  F = 5
5 P $^3/2$  F = 4
5 S $^1/2$  F = 3

780 nm
63 P $^3/2$  F = 4

776 nm

780 and 776 counterpropagating
780 and 1256 copropagating

Figure 6. Absorption signal from the 63P$^3/2$ Rydberg state in a room-temperature gas cell obtained with a three diode laser cascade setup. The 1256 nm laser is applied copropagating with the 780 nm laser.
1: Coherent excitation of Rydberg states

• overview

• calculation of rates

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• 3 photon methods

• experiments, coherent oscillations
Experiments

First experiments were in 1970s, 1980s, for a review see


Excited state absorption spectroscopy of alkaline earths selectively pumped by tunable dye lasers
I. Barium arc spectra

D J Bradley, P Ewart, J V Nicholas and J R D Shaw
Department of Pure and Applied Physics, The Queen’s University of Belfast
Belfast BT7 1NN, Northern Ireland

1973 data, resolution about 0.1 nm
Stark structure of the Rydberg states of alkali-metal atoms

Myron L. Zimmerman, Michael G. Littman, Michael M. Kash, and Daniel Kleppner

Experiments

1979 data, resolution about 10 GHz
2 photon excitation – Autler Townes splitting

2003 data, resolution few MHz,
Note, Rabi frequency depends on ground quantum state.
2 photon excitation – Rabi oscillations

Rabi Oscillations and Excitation Trapping in the Coherent Excitation of a Mesoscopic Frozen Rydberg Gas

M. Reetz-Lamour, T. Amthor, J. Deiglmayr, and M. Weidemüller*

FIG. 1 (color online). (a) Relevant level scheme for two-photon excitation of rubidium. (b) Preparation of a mesoscopic subensemble. All atoms are pumped to the $F = 1$ hyperfine component of the ground state. Only a tight tube of atoms is pumped to $F = 2$ whence the excitation originates. Within this tube, a mesoscopic subensemble containing about 100 atoms is excited to Rydberg states with two counterpropagating laser beams at 780 and 480 nm. The latter has a flattop intensity profile shown in the inset, to ensure a constant Rabi frequency.

FIG. 2. Rabi oscillations between the $5S_{1/2}$ and $31D_{5/2}$ states of $^{87}$Rb. Each dot is an average of measurements over 28 experimental repetition cycles. The solid line shows the simulated excitation probability taking the measured intensity distribution, the residual $5P_{3/2}$ admixture, and our finite laser linewidth into account. The scaling between the detector signal and simulated excitation probability is a free parameter that represents the detector efficiency.
Rabi oscillations in hot vapor cells

Oscillations on nsec time scale!
Single atom Rabi oscillations

Rabi Oscillations between Ground and Rydberg States with Dipole-Dipole Atomic Interactions


$\Omega/2\pi = 0.36 \text{ MHz}$

$t_\pi = 1.4 \mu s$

amplitude = 0.96

Coherent excitation of a single atom to a Rydberg state

Y. Miroshnychenko, A. Gaéan, C. Evelin, P. Grangier, D. Comparat, P. Pillet, T. Wilk, and A. Browaeys

FIG. 4. (Color online) Coherent Rydberg excitation of a single atom to the level $|5S_{1/2}, F = 3, m_F = 3 \rangle$ for different experimental parameters: (a) $(\Omega_\theta, \Omega_\alpha, \Delta)/2\pi = (255, 24, 400) \text{ MHz}$, (b) $(\Omega_\theta, \Omega_\alpha, \Delta)/2\pi = (250, 28, 600) \text{ MHz}$, and (c) $(\Omega_\theta, \Omega_\alpha, \Delta)/2\pi = (80, 70, 600) \text{ MHz}$. Each point corresponds to 100 repetitions of the experiment. The blue line is the result of a Monte Carlo simulation of the dynamics of a five-level system, which includes a decay from the intermediate state, fluctuations of the power and of the frequency of the lasers, and imperfection of the optical pumping. The dotted orange line shows for comparison the results of simulations with the same parameters of $\Omega_\theta$, $\Omega_\alpha$, and $\Delta$ but without fluctuations.
Experimental approach - UW Madison

Reference cavity is 5 kHz linewidth, ULE spacer Fabry-Perot in temperature stabilized vacuum can. This gives long term stability < 100 kHz and ~200 Hz linewidths. Verified by beating two lasers together.

State preparation and selection

High fidelity oscillations require optical pumping to a single initial state and Rydberg state selection.

We start by pumping into $f=2, m_f=0$.

This hyperfine state is a superposition of electron spin up and spin down.

The hyperfine interaction is negligible at $n=100$. With two $\sigma_+$ photons we can excite $97d_{5/2}$ $m_j=3/2$ or $m_j=5/2$.

The Rabi frequency is different for these two end states which leads to non-sinusoidal oscillations.
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The Rabi frequency is different for these two end states which leads to non-sinusoidal oscillations.
Coherence of Rydberg excitation

$|r\rangle \rightarrow \delta \rightarrow r \rightarrow \gamma$

$\omega_2 \rightarrow \Omega_2$

$p \rightarrow \Delta_p \rightarrow \gamma_p$

$|s\rangle \rightarrow \omega_1 \rightarrow \Omega_1$

$\theta = \Omega t$

$97d_{5/2}$

1.01 PHz

$5s_{1/2}$

initialize

$\pi/2$

$t$

$\pi/2$

measure
Coherence of Rydberg excitation

$\omega_2 - \omega_1 = \gamma_p$

$\gamma = \gamma_p$

$T_2 = 3.6 \mu s$

$97d_{5/2}$

$1.01 \text{ PHz}$

$5s_{1/2}$
Coherence of Rydberg excitation

Signal decays as

\[ e^{-t^2 / T_2^2} \]

- \( T_{2, \text{magnetic}} = 6.8 \text{ \mu s} \)
- \( T_{2, \text{Doppler}} = 4.2 \text{ \mu s} \)
Summary

• Excitation of Rydberg states can be performed coherently with modern, stabilized laser sources.

• I have concentrated on the simplest case of constant in time, linear excitation of non-interacting atomic samples.

• With interactions blockade plays an important role, leading also to a $N^{1/2}$ speedup.

• Quantum interference effects can be exploited to achieve large optical nonlinearities (C. Adams lectures)

• Temporally modulated pulse sequences such as STIRAP can lead to novel quantum states in interacting samples (K. Mølmer lectures)
II: Rydberg atom interactions

- orders of magnitude
- dipole-dipole interactions: resonant limit, van der Waals limit
- angular dependence
- Ground – Rydberg interaction
- ground state dressing
Long range interactions

Rb-Rb ground state magnetostatic interaction

$\Delta E \sim 100 \mu\text{Hz}$
Long range interactions

Rb-Rb ground state magnetostatic interaction

$\Delta E \sim 100 \, \mu\text{Hz}$

Rydberg $n=100$ van der Waals interaction

$\Delta E \sim 100 \, \text{MHz}$

12 orders of magnitude!
Rydberg interactions: strong and controllable

\[ U(\text{Hz}) \sim \frac{1}{R^6} \]
Rydberg interactions: strong and controllable

![Graph showing Rydberg interactions](image)
Rydberg interactions: strong and controllable

\[ U(\text{Hz}) \]

- d-d $\sim 1/R^3$
- mag. d-d $\sim 1/R^3$
- vdw $\sim 1/R^6$

Rydberg 100s

12 orders of magnitude

R (\mu m)

\[ 10^{-9} \]
\[ 10^{-5} \]
\[ 10^{-3} \]
\[ 10^{1} \]
\[ 10^{7} \]
\[ 10^{11} \]
Rydberg interactions: strong and controllable
Rydberg - Rydberg interaction

Förster resonance

\[ |n,p\rangle \quad |n,s\rangle \quad |n-1,p\rangle \]


Rydberg theory:

Protsenko, Reymond, Schlosser, Grangier PRA 2002
Walker & MS, JPB 2005, PRA 2008
Li, Tanner, Gallagher PRL 2005
**Rydberg - Rydberg interaction**

**Förster resonance**

![Diagram of Förster resonance](image)


Rydberg theory:

Protsenko, Reymond, Schlosser, Grangier PRA 2002
Walker & MS, JPB 2005, PRA 2008
Li, Tanner, Gallagher PRL 2005
Rydberg - Rydberg interaction

Förster resonance

\begin{align*}
|n,p\rangle & \quad \rightarrow \quad |n-1,p\rangle \\
|n,s\rangle & \quad \rightarrow \quad |n-1,s\rangle
\end{align*}

Th. Förster,
Zwischenmolekulare energiewanderung
und fluoreszenz,
Annalen der Physik 2, 55 (1948).

Rydberg theory:

Protsenko, Reymond, Schlosser, Grangier PRA 2002
Walker & MS, JPB 2005, PRA 2008
Li, Tanner, Gallagher PRL 2005
Resonant d-d to van der Waals

The resonant interaction strength is

\[ V_{dd} \sim C_3/R^3 \quad C_3 \sim d_1 d_2 \]

With energy defect \( \delta \) we get

\[ V_{vdW} \sim C_6/R^6 \quad C_6 = C_3^2/h\delta \]
Molecular potentials

Dipole-dipole operator in spherical basis

\[
\hat{V}_{dd} = -\frac{e^2}{4\pi\epsilon_0 r^3} \sqrt{6} \sum_p C^{20}_{1p1-p} r_A p r_B p
\]

\(M = m_A + m_B\) is conserved

Symmetrized states

\[
\begin{align*}
|1\rangle &= |n_00; n_00\rangle, & |3\rangle &= |n_{11}; n_{21} - 1\rangle, \\
|2\rangle &= |n_{10}; n_{210}\rangle, & |4\rangle &= |n_{11} - 1; n_{211}\rangle
\end{align*}
\]

Hamiltonian

\[
\mathcal{H}_0 = \hbar \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \delta & 0 & 0 \\
0 & 0 & \delta & 0 \\
0 & 0 & 0 & \delta
\end{pmatrix}, \quad \mathcal{H}_1 = -\frac{\sqrt{2}}{3} U_3 \begin{pmatrix}
0 & 2 & 1 & 1 \\
2 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}
\]

energy defect

\[
\hbar \delta = E(n_1p) + E(n_2p) - 2E(ns)
\]

long range interaction

\[
U_3(r) = \frac{e^2}{4\pi\epsilon_0 r^3} \langle n_{11} || r || n_0 \rangle \langle n_{21} || r || n_0 \rangle
\]
Eigenvalues - van der Waals limit

\[ U_\pm(r) = \frac{\hbar \delta}{2} \pm \sqrt{\frac{(\hbar \delta)^2}{4} + \frac{4U_3^2(r)}{3}} \]

\[ \hbar \delta = E(n_1p) + \tilde{E}(n_2p) - 2E(ns) \]

\[ U_3(r) = \frac{e^2}{4\pi \varepsilon_0 r^3} \langle n_31||r||n0 \rangle \langle n_41||r||n0 \rangle \]

When the energy defect is large \( \hbar \delta >> U_3 \).

\[ U_\pm(r) \approx \frac{\hbar \delta}{2} \pm \frac{4U_3^2(r)}{3\hbar \delta} \]

We recover the short range van der Waals interaction

\[ U_{vdW} = \frac{C_6}{r^6} = \frac{4U_3^2(r)}{3\hbar \delta} \]

For Rb, Cs large \( n \), \( \nu_s - \nu_p \sim 0.5 \).
This implies \( \delta \sim 1/n^4 \) so \( C_6 \sim n^{11} \)

\[ C_6 \sim n^{12} \] heavy alkali ns states
When the energy defect is small $\hbar \delta \ll U_3$ we obtain a long range $1/r^3$ interaction

$$U_\pm(r) \approx \frac{\hbar \delta}{2} \pm \frac{2}{\sqrt{3}} U_3(r)$$

$$\frac{C_3}{r^3} = \frac{2}{\sqrt{3}} U_3(r)$$

The effective dipole-dipole interaction is isotropic (for s states).
This simplest case $ns+ns \rightarrow np + n''p$ is easily solved by hand.

There are many more cases “channels”.

---

TABLE I. Relative interaction strengths for van der Waals interactions of Rydberg atoms, for various collision channels. The potential energy at distance $R$ is the product $C_6 D_0 / R^6$, which contains the effects of Zeeman degeneracy, with the overall $C_6$ coefficient [Eq. (37)] for a particular channel that depends only on the energy level structure and radial matrix elements. Cases where the $j$ quantum number is not included in the channel description are the sum over fine-structure components of the final state.

| Channel          | $|M| \{D_0\}$   | Channel          | $|M| \{D_0\}$   |
|------------------|----------------|------------------|----------------|
| $s_{1/2} + s_{1/2}$ | 1 \{1.33\} | $s_{1/2} + s_{1/2}$ | 1 \{0.0988\} |
| $p + p$          | 0 \{1.33,1.33\}| $p_{1/2} + p_{1/2}$ | 0 \{0.395,0\} |
| $s_{1/2} + s_{1/2}$ | 1 \{0.346\} | $s_{1/2} + s_{1/2}$ | 1 \{0.543\} |
| $p_{1/2} + p_{3/2}$ | 0 \{0.444,0.0494\} | $p_{3/2} + p_{3/2}$ | 0 \{0.84,0.444\} |
| $p_{1/2} + p_{1/2}$ | 1 \{0.0988\} | $s_{1/2} + d_{3/2}$ | 0 \{0.444,0.0494\} |
| $s_{1/2} + s_{1/2}$ | 0 \{0.395,0\} | $p_{3/2} + p_{3/2}$ | 3 \{0\} |
| $p_{3/2} + p_{1/2}$ | 1 \{0.543\} | $s_{1/2} + s_{1/2}$ | 2 \{0,0\} |
| $d_{3/2} + d_{3/2}$ | 0 \{0.84,0.444\} | $s_{1/2} + s_{1/2}$ | 1 \{0.543,0,0\} |
|                 |                | $s_{1/2} + d_{3/2}$ | 0 \{0.84,0.444,0,0\} |
| $p_{3/2} + p_{3/2}$ | 3 \{0\}   | $s_{1/2} + s_{1/2}$ | 2 \{0.267\} |
| $s_{1/2} + d_{3/2}$ | 2 \{0.08,0.00889\} | $p_{3/2} + p_{3/2}$ | 2 \{0.48,0.0533\} |
|                 | 1 \{0.0622,0.0491,0.00322\} | $s_{1/2} + d_{3/2}$ | 1 \{0.64,0.0533,0.16\} |
|                 | 0 \{0.0494,0.0178,0,0\} |                 | 0 \{0.693,0.267,0,0\} |

etc…
The higher angular momentum channels are generally anisotropic, and can have zero or small eigenvalues.

Walker & MS PRA (2008)

| Channel          | $|M| \{D_\phi\}$       | Channel          | $|M| \{D_\phi\}$       |
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| $p_{1/2} + p_{1/2}$ | $1 \{0.0988\}$          | $p_{1/2} + p_{1/2}$ | $1 \{0.346\}$         |
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| $p_{3/2} + p_{1/2}$ | $1 \{0.543\}$          | $p_{3/2} + p_{3/2}$ | $3 \{0\}$             |
| $d_{3/2} + d_{3/2}$ | $0 \{0.84, 0.444\}$    | $s_{1/2} + s_{1/2}$ | $2 \{0, 0\}$          |
| $P_{3/2} + P_{3/2}$ | $2 \{0.08, 0.00889\}$  | $s_{1/2} + d_{3/2}$ | $1 \{0.543, 0, 0\}$   |
| $s_{1/2} + d_{3/2}$ | $0 \{0.0494, 0.0178, 0, 0\}$ | $s_{1/2} + d_{3/2}$ | $0 \{0.693, 0.267, 0, 0\}$ |

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etc...
Förster zero states are linear combinations of Zeeman levels with zero coupling.

Consider $M = m_A + m_B = 0$. Say initial states have angular momentum $j$, there are $2j+1$ states with $M=0$.

These couple to $j_s, j_t$ with $j_s \leq j_t$, giving $2j_s + 1$ $M=0$ intermediate states.

We get zero vdW coupling to the Förster zero state $|\psi_F>\$ when

$$\langle st|V_{dd}|\psi_F\rangle = \sum_{J=0}^{2j} \langle st|V_{dd}|J\rangle \langle J|\psi_F\rangle = 0$$

for all $2j_s + 1$ coupled states.

This is $2j_s + 1$ equations for $2j+1$ unknowns. A Förster zero solution typically exists when $j_s < j$.

When $j_s = j$, no exact zero, but we always find very small eigenvalues.
Angular structure examples

The $43d_{5/2}$ state has a small defect and very strong interaction.

$$43d_{5/2} + 43d_{5/2} \rightarrow 45p_{3/2} + 41f$$

Energy defect 7.5 MHz.

There are Förster zero states, so strong angular dependence.

We need to go to 55s to get a comparable strength isotropic interaction.
Angular structure examples

States with $|l-l'|=1$ have a strong resonant interaction. If one of them is an $s$ state the interaction is largely isotropic.

We can also have large asymmetries between $s-s$, $s-p$ and $p-p$, interaction strengths. Very useful for various quantum gate protocols.

Discussion

This analysis assumes a single channel dominates. At high n and small separation this is not a good approximation.

There are 18 dipole coupled pairs of states within +/- 4 GHz of $60p_{3/2} + 60p_{3/2}$.

![Diagram showing two-atom energy levels connected to the $|60p_{3/2}60p_{3/2}\rangle$ state by the dipole-dipole interaction.](image)
Discussion

However, not all near resonant states play a role since the dipole matrix elements are concentrated at neighboring $n$, $n'$. 

**FIG. 4.** (Color online) Radial matrix elements divided by $n^2$ for transitions $ns_{1/2} \rightarrow np_{3/2}$ (filled circles) and $ns_{1/2} \rightarrow np_{1/2}$ (filled boxes) in Cs and Rb, in atomic units.
Discussion

When multiple channels play a role it is easiest to use direct numerical diagonalization of the Hamiltonian. Easy to include Zeeman, Stark effects.

\[ H = H_{\text{atomic}} + H_{dd} + H_{\text{Stark}} + H_{\text{Zeeman}} \]

Spaghetti physics ensues. It appears highly unlikely that Förster zero interactions persist.

Including enough states to ensure convergence is an open problem (I. Deutsch).
Calculating matrix elements

As for optical excitation there are several calculation methods available.

Coulomb wave functions (Seaton)
Model potentials


Numerics are more challenging than for excitation from ground state since both wavefunctions are spatially extended.

For excitation from ground state we had x2 discrepancy.

In this case WKB agrees very well with Coulomb wavefunctions.
Coulomb – WKB comparison

Radial matrix element
50s - n’ \( p_{1/2} \)

Error:
\[
\frac{R_C - R_{WKB}}{|R_C| + |R_{WKB}|}
\]
Ground-Rydberg interactions

- For $n \approx 100$ Rydberg-Rydberg interaction is stronger than ground-ground mag. dip. by $10^{12}$, ground-ground vdW by $10^{17}$ at 1 $\mu$m.

- What about Rydberg-ground? Guess $3 \times 10^8$ stronger than ground-ground vdW.

Two cases

Ground perturber inside Rydberg wavefunction - molecular binding (T. Pfau lectures)

perturber outside, $R > a_0 n^2$
Van der Waals interaction
Ground-Rydberg interactions

We can calculate using the same method as for Rydberg-Rydberg

\[ C_6(100s-5s) \sim 17 \text{ (Hz } \mu \text{m}^6) \]

\[ C_6(5s-5s) \sim 0.64 \times 10^{-6} \text{ (Hz } \mu \text{m}^6) \]

The Förster defect is essentially the 5s-5p splitting 12820 cm\(^{-1}\)

The ratio is \(0.27 \times 10^8\)

This interaction is of relevance to high density/high precision experiments.
Rydberg dressing of ground state interactions

If the excitation is off-resonant the ground state atoms will be “dressed” by the Rydberg interaction.

\[ \Delta \gg \Omega \]

original proposal:

Details for few atoms:

Relevant for nonlinear optics, atom-optics, Quantum simulation of spin models
Effective interaction

• To find the effective nonlocal interaction consider two-atoms in the basis

\[ \{ |gg\rangle, |gr\rangle, |rg\rangle, |rr\rangle \} \]

Interaction picture Hamiltonian

\[ \hat{H} = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega^* & \Omega^* & 0 \\ \Omega & -2\Delta & 0 & \Omega^* \\ 0 & 0 & -2\Delta & \Omega^* \\ 0 & \Omega & \Omega & 2(-2\Delta + \Delta_{Ry}) \end{pmatrix} \]

Eigenvalues give the effective interaction potential.

Soft core form due to blockade.
Effective interaction

We can find the soft core interaction strength without solving the Hamiltonian

\[ -\frac{|\Omega|^4}{8\Delta^3} \]

Two noninteracting atoms have

Due to blockade only get

Admixture of Rydberg in ground state

Interaction effect

\[ \Delta_{LS1} = \frac{|\Omega|^2}{4\Delta} \]

\[ \Delta_{LS2} = 2 \cdot \frac{|\Omega|^2}{4\Delta} \]

\[ \Delta_{LS1} - \Delta_{LS2} = -\frac{|\Omega|^2}{4\Delta} \]

Multiply

\[ \frac{|\Omega|^2}{2\Delta^2} \]

\[ -\frac{|\Omega|^4}{8\Delta^3} \]
Summary

- Rydberg states have extraordinarily strong interactions.
- The interaction can be transferred to the ground states by dressing.
- When one channel is dominant the problem can be solved more or less analytically.
- For multiple channels numerics are best suited.
- The Rydberg dipole-dipole interaction has been observed in experiments with many atoms: frozen Rydberg gas, line broadening, blockade (T. Gallagher lectures).
- The interaction is crucial for quantum gate experiments (O. Morsch lectures) and recent single photon experiments.
- A quantitative observation of the dipole-dipole shift at the level of two atoms remains an outstanding challenge.
III: Coherence properties of ground and Rydberg traps

- Optical traps and ground state (qubit) coherence
- Ramsey spectroscopy
- Magic traps
- Traps for Rydberg atoms
- 3D Rydberg atom trapping
Traps and coherence

Long coherence times compared to gate times or other dynamical times are important for studies of quantum dynamics.

Several decoherence mechanisms:
- collisional loss - about 50 s. lifetime at $10^{-10}$ mbar
- radiative decay
- light scattering
- magnetic noise
- electric fields
- AC electric field in optical traps
- motional dephasing

Types of traps:
- Electric
- Magnetic
- Optical
- Optical in cavities or near surfaces
Radiative decay

For optical qubits metastable upper level has a finite radiative lifetime, e.g. 160 s for Sr $^3P_0$

For hyperfine qubits the radiative lifetime is not of concern. For Cs the lifetime is

- $T=0$: 23,300 years
- $T=300$ K: 34 years

In a giga-qubit room temperature Cs computer 1 decay per second (QEC is needed!)
Magnetic decoherence

$^{87}$Rb $5S_{1/2}$

Ground state, single unpaired electron, $L=0$, $S=1/2$, $J=1/2$

Nucleus, $^{87}$Rb $I=3/2$, so total angular momentum $F=1,2$.

$$\hat{H} = A \hat{I} \cdot \hat{J}$$

$$\Delta U_{hf} = \frac{A}{2} [F(F+1) - I(I+1) - J(J+1)]$$

$$\Delta U_{F=2} = \frac{3A}{4}$$

$$\Delta U_{F=1} = -\frac{5A}{4}$$

$A = 3.417$ GHz
Breit-Rabi plot

\[ \hat{H} = A \mathbf{I} \cdot \mathbf{J} + \frac{\mu_B}{\hbar} \left( g_s \hat{S} + g_I \hat{L} + g_I \hat{I} \right) \cdot \mathbf{B} \]

\[ \Delta U_{F=2} = \frac{3A}{4} \]
\[ \Delta U_{F=1} = -\frac{5A}{4} \]
\[ A = 3.417 \text{ GHz} \]
Magnetostatic traps

There are weak (magnetic) field seeking ground states.

$^{87}\text{Rb} \ |1, -1>, \ |2, 1>, \ |2, 2>$

This is widely used for experiments with degenerate quantum gases, BEC and Fermi systems.
Fluctuation insensitive qubit trapping

$^{87}\text{Rb } 5S_{1/2}$

$F=2$

$|0\rangle$

$|1\rangle$

$m_F=0$

6.8 GHz

At zero B field $m=0$ Zeeman states are ideal qubits.

Bias field needed to define quantization axis.
Fluctuation insensitive qubit trapping

$^{87}\text{Rb } 5S_{1/2}$

$F=2$

$|1\rangle$

$|2,1\rangle - |1,-1\rangle$

3.236 G

6.8 GHz
Fluctuation insensitive qubit trapping

$^{87}\text{Rb} \; 5S_{1/2}^1 \quad F=2 \quad \rightarrow \quad F=1$

$6.8 \text{ GHz}$

$|2,-1\rangle - |1,0\rangle$

$653 \text{ G}$
Fluctuation insensitive qubit trapping

$^{87}\text{Rb}\ 5S_{1/2}$

F=2

1

$6.8\ \text{GHz}$

655 G
Fluctuation insensitive qubit trapping

$^\text{87}\text{Rb} \ 5S_{1/2}$

F=2

$^1$}

6.8 GHz

1220 G

$|2, -1\rangle - |1, -1\rangle$

1220 G

1218  1219  1220  1221  1222

B (G)

U (MHz)
Long coherence time magnetically trapped atoms

$^{87}\text{Rb } 5S_{1/2}$

$F=2$  

$F=1$

3.236 G bias field

6.8 GHz

Coherence in Microchip Traps

Philipp Treutlein, Peter Hommelhoff, Tilo Steinmetz, Theodor W. Hänsch, and Jakob Reichel

FIG. 1. Ramsey spectroscopy of the $|0\rangle \leftrightarrow |1\rangle$ transition with atoms held at a distance $d = 9 \ \mu\text{m}$ from the chip surface. An exponentially damped sine fit to the Ramsey fringes yields a $1/e$ coherence lifetime of $\tau_c = 2.8 \pm 1.6 \ \text{s}$. Each data point corresponds to a single shot of the experiment.
Magnetostatic trap arrays

Absorption image of the loaded lattice, showing ~500 traps loaded with 200–2500 atoms each

Periodicity about 20 μm

Microtrap arrays on magnetic film atom chips for quantum information science

V. Y. F. Leung · A. Tauschinsky · N. J. van Druten · R. J. C. Spreeuw

also, Hannaford, Hinds
Electrostatic traps

Cs has a dc polarizability $\alpha = +1.0 \times 10^{-5}$ Hz/(V/m)$^2$ giving a potential

$$U = -\frac{1}{2} \alpha E^2$$

Potential minimum at field maximum.

General property of harmonic functions – no maximum inside simply connected domain. (Earnshaw theorem)

No electrostatic traps for atomic ground states.
Optical traps for atoms

- Far off-resonance traps (FORTs): sub-micron localization, long coherence times.

  - red detuned
    \[ \Delta = \omega - \omega_a < 0 \]
  - blue detuned
    \[ \Delta = \omega - \omega_a > 0 \]

  + simple optics
  - more light scattering and differential light shifts
  + more complex optics
  + less light scattering and differential light shifts
  + Rydberg trapping
Light shifts – AC Stark effect

An off-resonant optical field shifts the energy of atomic levels. This is important for optical traps, Z gates, and decoherence.

Electric dipole interaction

\[ \hat{H} = -\hat{d} \cdot \mathbf{E} \]

To first order

\[ d_{gg} = \langle g | \hat{d} | g \rangle = 0 \]

However, if we go to second order in the field we get a nonzero \( d_{gg} \). Write this as \( d_{gg} \equiv d = \alpha \mathbf{E} \)

Then

\[ U_{ac} = \langle \hat{H} \rangle = -\langle \hat{d} \cdot \mathbf{E} \rangle = -\langle \hat{\alpha} \cdot \mathbf{E}^2 \rangle = -\frac{1}{4} \alpha \mathbf{E}^2 \]

with \( \alpha \) the polarizability.
Calculation of $\alpha$

Use second order perturbation theory:

$$U_{ac} = \sum_e \frac{\langle e | \hat{H} | g \rangle |^2}{U_g - U_e}$$

$$= \sum_e \frac{\mathbf{E} \cdot \langle g | \hat{d} | e \rangle \langle e | \hat{d} | g \rangle \cdot \mathbf{E}}{(\hbar \omega_g + \hbar \omega) - \hbar \omega_e}$$

The interaction involves two powers of the vector $\mathbf{d}$, so in general $\alpha$ is a 2$^{nd}$ rank tensor with scalar, vector, and tensor parts.

Let's just consider the scalar polarizability. Two-level model, RWA

$$U_{ac} = \frac{\hbar^2 |\Omega|^2}{4(\hbar \omega_g + \hbar \omega) - \hbar \omega_e} = \frac{\hbar |\Omega|^2}{4 \Delta}$$

$$\langle e | \hat{d} | g \rangle \cdot \mathbf{E} = \frac{\hbar \Omega}{2}$$

The polarizability is

$$U_{ac} = -\frac{1}{4} \alpha \mathcal{E}^2 \quad d^2 = \frac{3\pi \varepsilon_0 c^3 \hbar \gamma}{\omega_a^3}$$

$$\alpha_{\text{two-level}} = -\frac{3\pi \varepsilon_0 c^3 \gamma}{\omega_a^3} \frac{\gamma}{\Delta}$$
Light scattering

Photon scattering rate is

\[ r = \rho_{ee} \gamma \]

Excited state population

\[ \rho_{ee} = \frac{1}{2} \frac{I/I_s}{1 + 4 \frac{\Delta^2}{\gamma^2} + \frac{I}{I_s}} \]

so

\[ r = \frac{\gamma}{2} \frac{I/I_s}{1 + 4 \frac{\Delta^2}{\gamma^2} + \frac{I}{I_s}} \to \frac{\gamma}{8(\Delta^2/\gamma^2)} \frac{I}{I_s} \] (large detuning)

We see that

trap depth \( \sim \frac{1}{\Delta} \) scattering rate \( \sim \frac{1}{\Delta^2} \)

At large detunings we have near ideal conservative traps.
If we store a hyperfine qubit in an optical trap, Rayleigh scattering gives heating but largely preserves the quantum state. Raman scattering changes the quantum state.

The amplitudes for Rayleigh scattering add. Rayleigh rate $\sim 1/\Delta^2$.

For Raman scattering ($f_1$ to $f_2$) they cancel giving Raman rate $\sim 1/\Delta^4$.

Qualitative explanation. At large detuning compared to the fine structure splitting, the alkali atom looks like a spin $\frac{1}{2}$ particle, no tensor polarizability quantum state is preserved.

Also require linear light polarization to suppress vector polarizability.
This was accepted truth for 15 years. However, the Bollinger group showed that also Rayleigh scattering can lead to decoherence scaling as $1/\Delta^2$ in some cases.
Optical trapping

Experimental Observation of Optically Trapped Atoms

Steven Chu, J. E. Bjorkholm, A. Ashkin, and A. Cable

AT&T Bell Laboratories, Holmdel, New Jersey 07733

We report the first observation of optically trapped atoms. Sodium atoms cooled below $10^{-3}$ K in "optical molasses" are captured by a dipole-force optical trap created by a single, strongly focused, Gaussian laser beam tuned several hundred gigahertz below the $D_1$ resonance transition. We estimate that about 500 atoms are confined in a volume of about $10^3 \mu m^3$ at a density of $10^{11} - 10^{12}$ cm$^{-3}$. Trap lifetimes are limited by background pressure to several seconds. The observed trapping behavior is in good quantitative agreement with theoretical expectations.

FIG. 2. (a) Photo showing the collimating nozzle, atomic beam, and atoms confined in OM. The distance from the nozzle to the OM region is 5 cm. (b) Photo taken after the atomic source and the slowing laser beam have been turned off, showing trapped atoms.
Single Atoms in an Optical Dipole Trap: Towards a Deterministic Source of Cold Atoms

D. Frese, B. Ueberholz, S. Kuhr, W. Alt, D. Schrader, V. Gomer, and D. Meschede
Optical trapping – single atom blockade

Sub-poissonian loading of single atoms in a microscopic dipole trap

Nicolas Schlosser, Georges Reymond, Igor Protosenko & Philippe Grangier

NATURE | VOL 411 | 28 JUNE 2001
Optical trapping – 91% atom loading

arXiv:1208.0707v1

High efficiency preparation of single trapped atoms using blue detuned light assisted collisions

A. V. Carpentier,¹ Y. H. Fung,¹ P. Sompet,¹ A. J. Hilliard,¹ T. G. Walker,² and M. F. Andersen¹
Spatial Localization

- With a single focused laser beam:

\[ \lambda_{D2} = 0.78 \, \mu m \]

\[ \lambda \approx 1 \, \mu m \]

\[ 2w \approx 5 \, \mu m \]

\[ T_a \approx 50 \, \mu K \]

\[ \Delta x \approx w(T_a/T_f)^{1/2} \approx 0.5 \, \mu m \]

\[ 2\sigma_x \approx 1 \, \mu m \]

\[ 2\sigma_z \approx 10 \, \mu m \]

\[ w = 3.0 \, \mu m, \quad \lambda = 1.06 \, \mu m, \quad P = 300 \, mW \]

\[ T_a = 0.1 - 0.5 \, mK, \quad U_f / k_B = 5 \, mK \]

Spatial variance

\[ \langle x_a^2 \rangle = \langle y_a^2 \rangle = \frac{w_{f0}^2}{4} \frac{T_a}{|U_m|} \]

\[ \langle z_a^2 \rangle = \frac{\pi^2 w_{f0}^4}{2\lambda_f^2} \frac{T_a}{|U_m|} \]

Fluorescence image
Blue detuned traps

- A large range of optical configurations have been used
- 3D optical lattices trap atoms at nodes of the field.
- Bottle Beam traps (BBT) only require access from a single side
Crossed vortex Bottle Beam Trap (BBT)
Atomic qubits in a BBT

Crossed vortex bottle beam trap for single-atom qubits

G. Li, S. Zhang, L. Isenhower, K. Maller, and M. Saffman

March 1, 2012 / Vol. 37, No. 5 / OPTICS LETTERS

- Lifetime 6s.
- Coherence time 43 ms.

Single atom detection

Lifetime 6s.

Coherence time 43 ms.
Optical resonators can serve as buildup cavities to enhance the field strength giving strong trapping with low optical power.

Evanescent fields at surfaces have large gradients giving good localization.

### Optical microcavities

*Kerry J. Vahala*  
*NATURE | VOL 424 | 14 AUGUST 2003*

<table>
<thead>
<tr>
<th></th>
<th>Fabry-Perot</th>
<th>Whispering gallery</th>
<th>Photonic crystal</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Q</td>
<td>$Q: 2.000$</td>
<td>$Q: 12.000$</td>
<td>$Q: 13.000$</td>
</tr>
<tr>
<td>$V_1$: 5</td>
<td>$(\lambda/n)^3$</td>
<td>$V_1$: 6 $(\lambda/n)^3$</td>
<td>$V_1$: 1.2 $(\lambda/n)^3$</td>
</tr>
<tr>
<td></td>
<td>$Q_{\text{res}}$: 7.000</td>
<td>$Q_{\text{poly}}$: 1.3x10^5</td>
<td></td>
</tr>
<tr>
<td>Low Q</td>
<td>$F: 4.8x10^5$</td>
<td>$Q: 8x10^9$</td>
<td>$Q: 10^6$</td>
</tr>
<tr>
<td>$V_1$: 1.690 $\mu m^3$</td>
<td>$V_1$: 3.000 $\mu m^3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
III: Coherence properties of ground and Rydberg traps

- Optical traps and ground state (qubit) coherence
- Ramsey spectroscopy
- Magic traps
- Traps for Rydberg atoms
- 3D Rydberg atom trapping
Qubit coherence – Ramsey measurement

The envelope of the Ramsey fringes is related to the coherence time $T_2$. Simply define $T_2$ as the $1/e$ time.
Ramsey measurement of decoherence

Start in $|1\rangle$, $\pi/2$ pulse

$|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + i|r\rangle)$

wait time $t$

$r\rangle$ picks up a phase of $\varphi = \varphi_R + \varphi_{st}$

$\pi/2$ pulse

$|\psi\rangle = \frac{1}{2}(|1\rangle + i|r\rangle) + \frac{e^{i\varphi}}{2}(i|r\rangle - |1\rangle)$

Probability of measuring $|1\rangle$

$P_1 = |\langle 1|\psi\rangle|^2 = \frac{1}{4}|1 - e^{2\varphi}|^2 = \frac{1}{2}(1 - \cos \varphi)$

Oscillation amplitude

$\frac{1}{2}(1 - \cos(\pi + \varphi_{st})) - \frac{1}{2}(1 - \cos(\varphi_{st})) = \cos(\varphi_{st})$

Zero mean random process

$\langle \cos(\varphi_{st}) \rangle = \langle e^{i\varphi_{st}} \rangle$
Magnetic decoherence

Magnetic phase
\[ \varphi_{st} = \nu_{B,g} \delta B t \]
\[ \nu_{B,g} = \frac{1}{\pi^2} \frac{\mu_B^2 B_0}{\hbar^2 \nu_{hf}} \text{ Hz/T.} \]

For \(^{87}\text{Rb}\) at \(B_0 = 0.5(3.5)\) G we find \(\nu_{B,g} = 0.57(4.0)\) Hz/mG.

Assume Gaussian fluctuations
\[ P(\delta B) = \left( \frac{1}{2\pi \sigma^2} \right)^{1/2} e^{-\delta B^2 / 2\sigma^2} \]

Average
\[ \langle e^{\nu_{\varphi_{st}}} \rangle = e^{-\nu_{B,g} \sigma^2 t^2 / 2} = e^{-t^2 / T_{2,gB}^2} \]

1/e time is
\[ T_{2,gB} = \frac{2^{1/2}}{\nu_{B,g} \sigma} \]
Motional decoherence

This is due to the different trap depths seen by the two hyperfine states.

The effect scales with the ratio of the differential light shift to the average shift.

The analysis is more complicated than for magnetic decoherence. Need to average over the motional states in the harmonic trap.

Find Ramsey envelope

$$\alpha(t) = \frac{1}{[1 + 0.95 \left( \frac{t}{T_{2,L}} \right)^2]^{3/2}}$$

$$T_{2,L} = 0.97 \frac{2\hbar}{\eta k_B T}$$

Kuhr, et al. PRA 72, 023406 (2005)

This more complicated behavior is well approximated by a Gaussian

$$e^{-t^2/T_{2,L}^2}$$
Extracting T2

Decoherence due to magnetic field and differential light shift model as: $e^{-t^2/T_2^2}$

two sources of decoherence

$$T_2 = \frac{T_{2,a} T_{2,b}}{(T_{2,a}^2 + T_{2,b}^2)^{1/2}}.$$

Matching to theory for T2 implies

|$\delta B_{\text{rms}} = 50 \text{ mG}$
|$T_a = 85 \mu\text{K}$
III: Coherence properties of ground and Rydberg traps

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State insensitive or “magic” trapping

Quantum State Engineering and Precision Metrology Using State-Insensitive Light Traps

Jun Ye,1* H. J. Kimble,2 Hidetoshi Katori3

27 JUNE 2008 VOL 320 SCIENCE

Fig. 2. 87Sr lattice clock. Blue laser light (1S0 to 1P1) is used to cool and trap Sr atoms at the center of the vacuum chamber. Atoms are further cooled with red light (1S0 to 3P1) in the second stage. Atoms are then loaded into a state-insensitive, vertical 1D optical lattice made of near-infrared light. (Top Right) Schematic levels for lattice spectroscopy, where the two electronic states are convolved with the quantized motional states. (Bottom Right) Line shape of a saturated 1S0 to 3P0 electronic transition and the motional sidebands.

Rempe, Kimble,...
Magic trapping for s and p states

Cs levels
Magic qubit trapping

- **Rydberg gates**
  - Red detuned trap laser
  - Blue detuned trap laser

- **Cooling, Measurement, gates**

- **Qubit**
  - Qubit levels get closer

\[ \delta \omega_{hf} \sim U_{trap} \frac{\omega_{hf}}{\Delta} \]
Magic qubit trapping

Rydberg gates  \ket{r}

cooling, Measurement, \ket{p}

gates

qubit  \ket{1}, \ket{0}

To make the qubit frequency independent of trap intensity tune in between the levels.

Small detuning - large scattering rate, but only need small intensity to compensate trap.
Suppression of inhomogeneous broadening in rf spectroscopy of optically trapped atoms

Ariel Kaplan, Mikkel Fredslund Andersen, and Nir Davidson

FIG. 1. Ground-level energies (a) and energy difference (b) of atoms trapped in a focused Gaussian beam. When exposed to the trapping laser, the two hyperfine levels have a different ac Stark shift (dashed line). An additional weak laser, detuned to the middle of the hyperfine splitting, creates an ac Stark shift (dotted line) such that the total amount of light shift (full line) is identical for both hyperfine levels.
“Doubly Magic” Conditions in Magic-Wavelength Trapping of Ultracold Alkali-Metal Atoms

Andrei Derevianko*  

Qubit hyperfine transition insensitive to both magnetic and intensity fluctuations!

TABLE I. Values of magic $B$ fields and magic wavelengths.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$B_m$, Gauss</th>
<th>$\lambda_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{87}\text{Rb}, I = 3/2, \nu_{\text{clock}} = 6.83 \text{ GHz}$</td>
<td></td>
<td>806 nm$^a$</td>
</tr>
<tr>
<td>$</td>
<td>2, 1\rangle \rightarrow</td>
<td>1, -1\rangle$</td>
</tr>
<tr>
<td>$^{85}\text{Rb}, I = 5/2, \nu_{\text{clock}} = 3.04 \text{ GHz}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>3, 1\rangle \rightarrow</td>
<td>2, -1\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>3, 2\rangle \rightarrow</td>
<td>2, -2\rangle$</td>
</tr>
<tr>
<td>$^{133}\text{Cs}, I = 7/2, \nu_{\text{clock}} = 9.19 \text{ GHz}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>4, 1\rangle \rightarrow</td>
<td>3, -1\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>4, 2\rangle \rightarrow</td>
<td>3, -2\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>4, 3\rangle \rightarrow</td>
<td>3, -3\rangle$</td>
</tr>
</tbody>
</table>

$^a$Nearly doubly magic.
Differential Light-Shift Cancellation in a Magnetic-Field-Insensitive Transition of $^{87}$Rb


The precise measurement of transition frequencies of trapped atomic samples is susceptible to inaccuracy arising from the inhomogeneous differential shift of the relevant energy levels in the presence of the trapping fields. We demonstrate near-complete cancellation of the differential ac Stark shift (“light shift”) of a two-photon magnetic-field-insensitive microwave hyperfine (clock) transition in $^{87}$Rb atoms trapped in an optical lattice. Up to 95(2)% of the differential light shift is cancelled while maintaining magnetic-field insensitivity. This technique should have applications in quantum information and frequency metrology.
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Traps for Rydberg atoms

- Many experiments with cold Rydberg atoms would be enhanced if we could trap Rydberg states:
  - Quantum gates, Rydberg-ground dressing, precision atomic measurements, Casimir-Polder studies,..

- Electrostatic and magnetostatic Rydberg trapping has been demonstrated.


- We use light for excitation, so optical traps would be convenient.
Optical Traps for Rydberg atoms

- Core polarizability is negligible.
- Rydberg polarizability is that of a free electron.
- To trap Rydberg atom need blue detuned trap.
- Choose wavelength and trap size so ground-Rydberg potentials are matched.
Ground-Rydberg magic trapping

\[ \alpha_e = -\frac{e^2}{m_e \omega^2} \]

\[ \alpha_0(50d) \]

\[ \alpha_0(6s_{1/2}) \times 10 \]

\[ \alpha_1(6s_{1/2}) \]

\[ \text{polarizability} (\text{Å}^3) \]

\[ \text{wavelength (μm)} \]
Ground-Rydberg magic trapping

Delocalized Rydberg wavefunction sees different intensity than ground state atom.

Calculate effective light shift with $|\psi|^2$ weighting of optical intensity
Ground-Rydberg magic trapping

Any wavelength for which ground and Rydberg states have negative $\alpha$ and $|\alpha_g| > |\alpha_{Ryd}|$ can be used.

**Graph:**

(Plot showing polarizability as a function of wavelength for $^{133}$Cs, with transitions labeled such as $6s-7p$, $6p_{3/2} \rightarrow 5d_{5/2}$.

**References:**

PHYSICAL REVIEW A 84, 043408 (2011)

Magic-wavelength optical traps for Rydberg atoms

S. Zhang, F. Robicheaux, and M. Saffman.
Ground-Rydberg magic trapping

We calculated light shifts for several BBT designs.

\[ \Delta E_R = \int d^3r U_p(\vec{r} + \vec{r}_i) |\psi_0^m(\vec{r}; \vec{R})|^2 \]
\[ = \frac{e^2}{2\varepsilon_0 c m_\epsilon \omega^2} \int d^3r I(\vec{R} + \vec{r}_i) |\psi_j^m(\vec{r}; \vec{R})|^2. \]

Adding a finite intensity at trap center gives ground – Rydberg matching
III: Coherence properties of ground and Rydberg traps

- Optical traps and ground state (qubit) coherence
- Ramsey spectroscopy
- Magic traps
- Traps for Rydberg atoms
- 3D Rydberg atom trapping
Observation of ponderomotive Rydberg potential

**State-Dependent Energy Shifts of Rydberg Atoms in a Ponderomotive Optical Lattice**

K. C. Younge,* B. Knuffman,† S. E. Anderson, and G. Raithel

**Trapping Rydberg Atoms in an Optical Lattice**

S. E. Anderson,* K. C. Younge, and G. Raithel

Red detuned trap is inverted during excitation.

Lattice induced shift of µwave spectra used to measure trapping.

This is a 2D trap.

FIG. 2 (color online). (a) Experimental microwave spectra for $P = 0.8$ W and optical pulse length $\tau_{ox} = 0.5$ $\mu$s for the indicated values of $\eta = P_{trans}/P$ (spectra offset for clarity). (b) Lattice potentials after inversion vs position for several of the $\eta$ values used in panel (a). The fully inverted case, $\eta = 1$, leads to the strongest blueshifted signal component, indicative of most efficient Rydberg-atom trapping.
Rydberg excitation in a BBT

Two photon excitation via 7p1/2 using 459 and 1038 nm lasers. Highly stabilized and referenced to frequency comb.

Data taken on 61d3/2 level.

Rabi oscillations

Rydberg data was taken while keeping the trap light on.
Rydberg trapping

1038 nm

$61d_{3/2}$

$\Gamma_{1038}$

$\Gamma_{\text{sp}}$

$\Gamma_{\text{trap}}$

P(ground)

Rate eq. model, global fit to three different pulse sequences gives

$\tau_{\text{sp}} = 79 \mu s$ (calculated)

$\tau_{1038} = 16 \mu s$

$\tau_{\text{trap}} = 390 \mu s$

Pulse time

strong evidence for Rydberg trapping

Detection efficiency

$16/79 = 20\%$

$P(r) = 0.8$
Rydberg trap lifetime

The trap lifetime is directly measured by blowing away non-Rydberg excited atoms.

\[ \tau = 17 \, \mu s \] (BBT off)

\[ \tau = 120 \, \mu s \] (BBT on)

3850 nm

61d_{3/2}
A Monte Carlo simulation including black body redistribution to other Rydberg levels (which are also trapped) gives a predicted lifetime before return to ground state of 143 µs.

This implies a trap lifetime

$$\tau_{\text{BBT}} > 750 \, \mu s.$$  

This data is preliminary.
Rydberg trap shift

Line center is about 130 kHz higher with trap on.

About 6.5 $\mu$K in temperature units.

We did not expect perfect magic trapping with this trap.

It should be possible to tune the trap shift close to zero in the future.
Summary

- Optical traps can hold neutral atoms with excellent coherence properties
- Rydberg atoms can be confined in bottle-beam type traps
- It is possible to design ground-Rydberg magic traps
- Experiments are showing clear evidence of Rydberg trapping
Rydberg atoms: excitation, interactions, trapping

I: Coherent excitation of Rydberg states

II: Rydberg atom interactions

III: Coherence properties of ground and Rydberg atom traps