Quantum gases and strongly correlated matter

Shannon Whitlock
University of Heidelberg, Germany

COHERENCE PISA SCHOOL 2012
http://coherence.physi.uni-heidelberg.de
whitlock@physi.uni-heidelberg.de

Intern:
coherence/cake

The experimental realization of Bose-Einstein condensation and Fermi-degeneracy ("quantum gases") in trapped atomic gases began a new frontier...

Atomic physics | Quantum optics | Many-body physics | Condensed matter | Quantum Information science

Until recently - interactions between particles played a relatively minor role (dilute gas regime)

$\Rightarrow$ Effective single-particle descriptions / mean field treatments

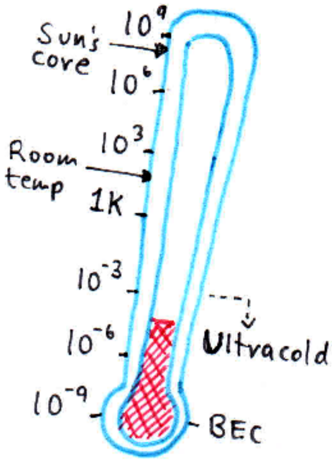
The new realm: **Strongly-interacting many-body quantum physics** $\Rightarrow$ Strongly correlated systems.

- Optical lattices
- One-dimensional systems
- Feshbach resonances
- Rydberg atoms

LECTURES:

I. Ultracold quantum gases
II. Rydberg interacting systems
III. Rydberg dressed quantum gases

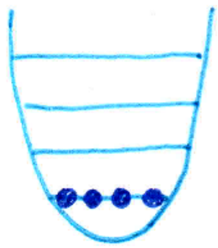
1.1 Ultracold atomic gases

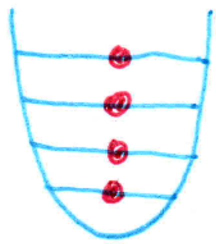


- Ultralow energy scales (nK regime $100\,nK\left(\frac{k_B}{h}\right) \approx 2\,kHz$)
- Physics dominated by quantum mechanical principles (s-wave scattering, quantum statistics, matterwaves, phase coherence ...)
- Precisely controlled systems
- Few- and many-body physics (Quantum chemistry, phase transitions)
- Quantum simulation and quantum information processing

The quantum regime: At low enough energy quantum statistics determines the behaviour of the gas.

BOSONS	FERMIONS
Bose-Einstein statistics (condensation)	Fermi-Dirac statistics (Pauli-Exclusion)





Short history:

1908 - Liquifaction of He @ 4.2K (birth of low-temperature physics)

1975 - Laser cooling proposed
T. Hänsch + A. Schawlow

1985/88 - 3D doppler cooling + Magneto optical trapping (1997 Nobel prize in physics) — S. Chu, W. Phillips, C. Cohen-Tannoudji

1995 — Bose-Einstein condensation (2001 Nobel prize in physics) — E. Cornell, C. Wieman, W. Ketterle

1999 - Fermi degeneracy

PRESENT — Strongly-correlated quantum systems

1.2 Laser cooling and trapping

Doppler cooling $\Rightarrow$ Slowing of atoms via light force

$v_{th} = \sqrt{k_B T / m}$ Accelleration

$v_{recoil} = \hbar k / m$ $\sim 10^4 - 10^5 g$

Due to the doppler shift an atom moving to the left can be made to interact more with light travelling to the right (red detuning)

Velocity drop from $\sim 100\,m/s$ $(\sim 300\,K)$ to $\sim cm/s$ $(\sim \mu K)$ over centimeters

Characteristic energy scale $E_{recoil} = \frac{\hbar^2 k^2}{2m}$ $\frac{E_{recoil}}{k_B} \sim \mu K$

Magnetic trapping

Exploit magnetic moment of neutral atoms

$$U = -\vec{\mu} \cdot \vec{B}$$

Low-field seeker
$\vec{\mu} \cdot \vec{B} < 0$

high-field seeker
$\vec{\mu} \cdot \vec{B} > 0$

× No field maxima in free space (Wing's theorem)

Adiabatic criteria

$$\frac{d\omega_L}{dt} \ll \omega_L^2$$

"Larmor freq."
$\omega_L = g_F m_F \mu_B |B| / \hbar$

Simplest stable magnetic trapping geometry: Ioffe-Pritchard trap

$$U(r) = \tfrac{1}{2} m \omega^2 r^2$$ ω: trapping frequency

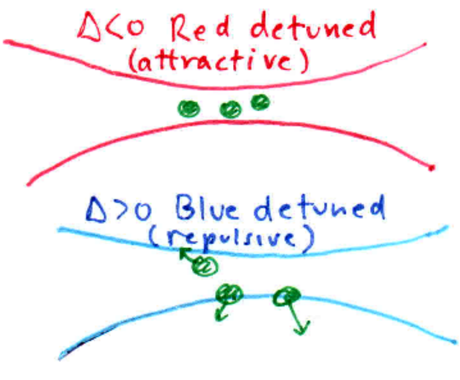
Optical dipole traps

Light induced electric dipole (AC Stark shift)

$$U_{dip} = \frac{1}{2\varepsilon_0 c} \mathrm{Re}(\alpha)\, I(r) \quad \propto \frac{\Gamma}{\Delta} I(r)$$

Scattering rate:

$$\gamma_{sc} = \frac{1}{\hbar \varepsilon_0 c} \mathrm{Im}(\alpha)\, I(r) \quad \propto \left(\frac{\Gamma}{\Delta}\right)^2 I(r)$$



Review article: R. Grimm, M. Weidemüller, Y. B. Ovchinnikov
Adv. At. Mol. Opt. Phys. 42 95 (2000)
arXiv:/physics/9902072

Evaporative cooling

Continuously reduce trap depth to remove most energetic atoms
$\Rightarrow$ RF spin flips or intensity ramp

Elastic collisions rethermalise gas to lower temperatures (nanoKelvin regime)

Review: W. Ketterle, N.J. van Druten
'Evaporative cooling of atoms'
Adv. At. Mol. Opt. Phys. 37 181 (1996)

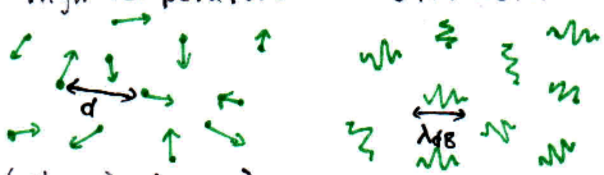
See slide

1.3 Bose-Einstein condensation (BEC)

de-Broglie wavelength $\lambda_{dB} = \left(\dfrac{2\pi\hbar^2}{m k_B T}\right)^{1/2}$

at 300K $\lambda_{dB} \sim 0.1\,\text{Å}$
100nK $\lambda_{dB} \sim 1\,\mu m$

high Temperature   ultracold



(classical gas)

Bose-Einstein condensation   Pure BEC

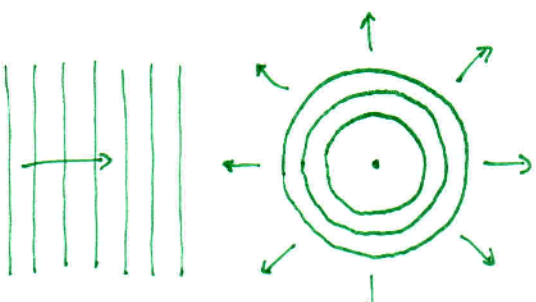
BEC: All particles occupy the same (single-particle) state

arXiv: cond-mat/9904034
'Making, probing and understanding Bose-Einstein condensates.'
W. Ketterle, D.S. Durfee
D.M. Stamper-Kurn

See slide

Ultracold interactions

At low energies the dominant interactions are two-body s-wave scattering



plane wave e^{ikz} spherical wave $e^{ik(r-a)}/r$

The precise shape of the short-range potential is not critical $\Rightarrow$ All relevant information is in a single parameter $\underline{a}$ "scattering length"

Can use an effective contact interaction

$$V = g\,\delta(r) \qquad g = \frac{4\pi\hbar^2 a}{m}$$

Scattering length is typically 10's of nanometers
but, it can sometimes be tuned $\Rightarrow$ Feshbach resonances

Theoretical description of BECs

Dilute gas regime: $na^3 \ll 1$

Time dependent Gross-Pitaevskii equation
(nonlinear Schrödinger equation)

$$i\hbar \frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + U(r) + g|\psi|^2\right)\psi \qquad \psi = \sqrt{n(r)}\, e^{i\phi(r)}$$

Effectively a single atom (mean field) description

- Collective excitations
- Vortices
- Matterwave interference + diffraction
- Multicomponent systems
- ...

$\boxed{\text{see slide}}$

1.4 Strongly-Correlated systems

Ratio of interaction energy I to Kinetic energy K: $\gamma = I/K \gg 1$

- Quantum magnetism
- Strongly coupled plasmas
- Unconventional superconductivity
- ...

} Quantum Simulation

$I = gn$

$K = \dfrac{\hbar^2}{m}\left(\dfrac{1}{\bar{r}^2}\right)$ Energy associated with localizing particles

$\bar{r}$ mean interparticle separation

In 3D: $\bar{r} = n^{-1/3}$ $\therefore$ $\gamma = 4\pi(na^3)^{1/3}$ Strongly interacting for high density

In 1D: $\bar{r} = n_{1D}^{-1}$ $\therefore$ $\gamma = \dfrac{m g_{1D}}{\hbar^2 n_{1D}}$ Strong interactions for low density !!

$\gamma \ll 1$

Superfluid

$\gamma \gg 1$

Tonks-Girardeau gas

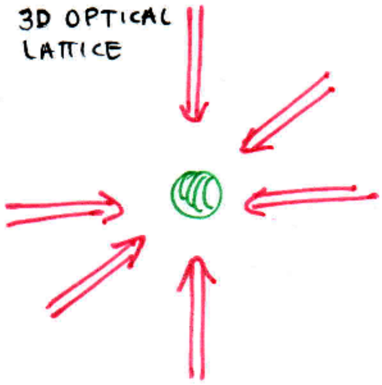
Expt: Kinoshita, Wenger + Weiss, Science 305 1125 (2004)
Paredes et al. Nature 429 277 (2004)

Superfluid-Mott insulator transition

Almost perfect realization of a quantum phase transition

Paradigm for the study of strongly-correlated systems

M. Greiner et al, Nature 415 39 (2002)



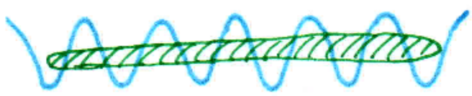
Bose-Hubbard Hamiltonian

$n_i = a_i^{+} a_i$

$$\hat{H} = \underbrace{-J\sum_{\langle ij\rangle} a_i^{+} a_j}_{\substack{\text{hopping term} \\ \text{(kinetic energy)}}} + \underbrace{\frac{U}{2}\sum_i n_i(n_i-1)}_{\substack{\text{on site interaction} \\ \text{(only for } n \geq 2)}} - \underbrace{\mu\sum_i n_i}_{\substack{\text{chemical} \\ \text{potential}}}$$

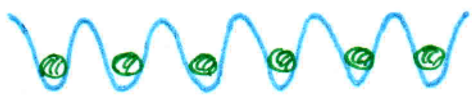
$J \gg U$: Superfluid phase

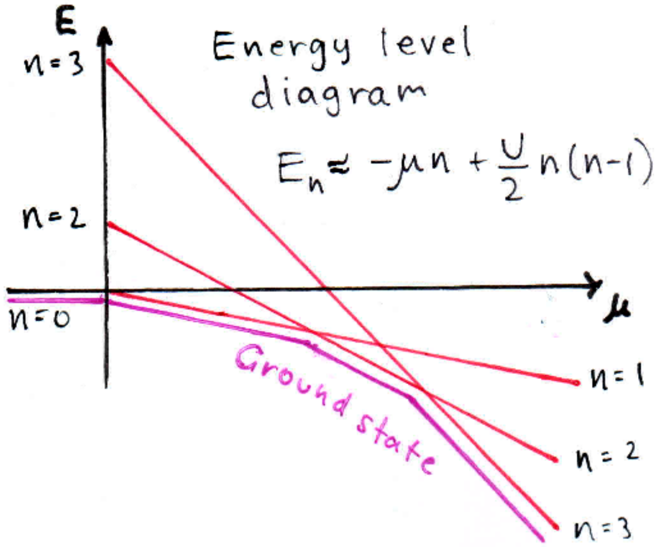
Poisson distribution for the number of atoms per site + Long range phase coherence.

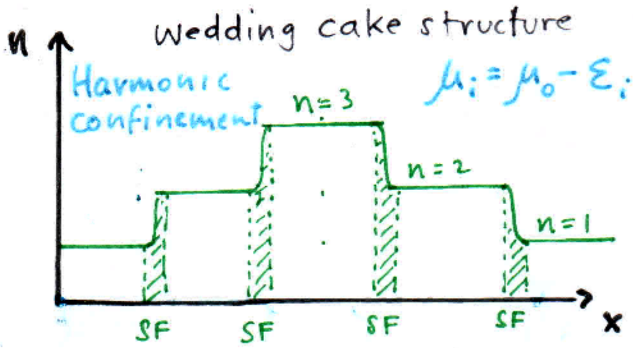


$U \gg J$: Insulating phase

Fixed number of particles per site + completely undefined phase

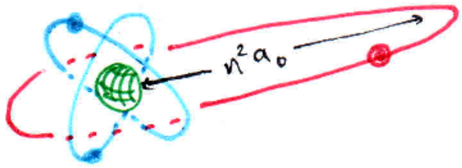




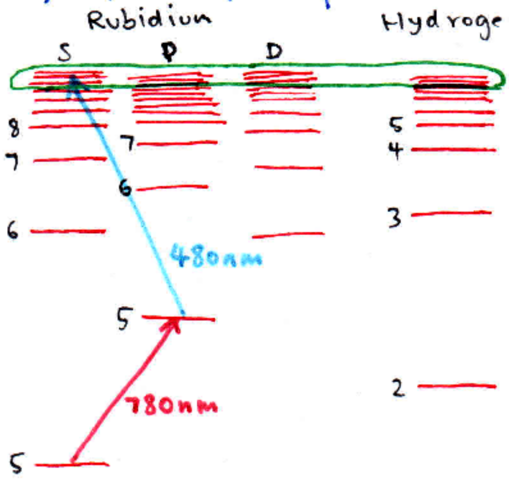


II. Rydberg interacting systems

2.1 Rydberg atoms



Atoms with one or more electrons excited to high-lying states (high principal quantum number $\underline{n}$)



Laser excitation usually couples to states with low angular momentum (S, P, D states)

Extreme properties

Principal quantum number: $n \gtrsim 20$ $\quad n^* \approx n - \delta_L$

$$E_{nL} = -R/(n^*)^2$$

Quantum defect
$\delta_{L=0} = 3.13$
$\delta_{L=1} = 2.64$
$\delta_{L=2} = 1.35$

Polarizability: For low L states (S, P, D)

$$\Delta E = \frac{1}{2} \alpha |F|^2$$ with α scaling as $(n^*)^7$

For scaling laws:
O'Sullivan + Stoicheff
PRA $\underline{31}$ 2718 (1985)
PRA $\underline{33}$ 1640 (1986)

Typical value for $n = 55$ state $\alpha = 100\,\mathrm{MHz}/(\mathrm{V/cm})^2$
i.e. 0.1 V/cm field gives energy shift ~ 1MHz

Lifetimes: Spontaneous emission + Blackbody redistribution

Beterov, PRA 052504 (2009)

For $n = 55$, lifetime is ~ 80μs (at room temp.)
Long for electronically excited states but short compared to typical timescales with ultracold atoms

Transition strength: $$\Omega = \frac{\mu}{\hbar}\left(\frac{2I}{c\varepsilon_0}\right)^{1/2}$$ μ = transition dipole moment

Deiglmayer et al, Opt. Comm. $\underline{264}$ 293 (2006)

$$\mu = \langle 5P_{3/2} | er | nL \rangle = C\,(n^*)^{-3/2}$$

$C_S = \frac{1}{\sqrt{3}}\, 4.508\, e a_0$

$C_D = \sqrt{2/5}\; 8.475\, e a_0$

Detection: Field ionization detection — State selective; Single atom sensitive; Position resolved

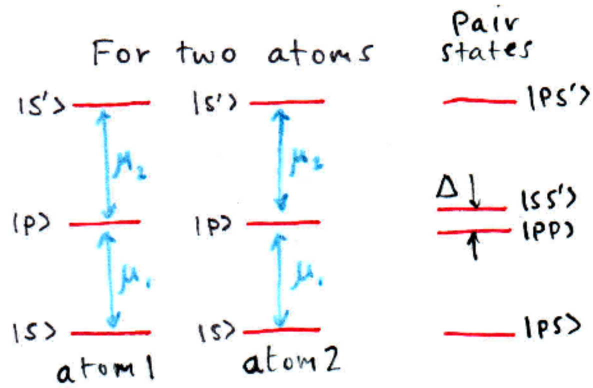
Optical (EIT)
J. Pritchard et al, PRL 105 193603 (2010)
G. Günter et al, PRL 108 013002 (2012)

$\boxed{\text{See slide}}$

Rydberg – Rydberg interactions

typical interparticle separation $\gg n^2 a_0$

Extremely strong interactions
- Dipolar interactions $\sim n^4/R^3$
- Van der Waals interactions $\sim n^{11}/R^6$



The dressed eigenenergies

$$E_\pm = \frac{-\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + \left(\frac{\mu_1\mu_2}{R^3}\right)^2}$$

Two regimes:

① $\Delta \to 0$ or $R \to 0$
$$E_\pm = \pm\frac{\mu_1\mu_2}{R^3} = \pm\frac{C_3}{R^3}$$

Δ can be tuned in an electric field "Förster resonance"

② $R \gg \left(\frac{C_3}{\Delta}\right)^{1/3}$
$$E = -\frac{\mu_1\mu_2/\Delta}{R^6} = -\frac{C_6}{R^6}$$

Scaling: μ scales with $(n^*)^2$ and Δ as $(n^*)^{-3}$

So, $C_3 \propto n^{*4}$ and $C_6 \propto (n^*)^{11}$

Sign of the vdW interaction depends on Δ
(for s-states $\Delta < 0$: purely repulsive)

Typical strength: $50p + 50p$ (Förster resonance) $C_3 \approx 13\,\mathrm{GHz}\,\mu m^3$
$\approx 1\,\mathrm{MHz}$ at $R = 20\,\mu m$ — van Ditzhuijzen, PRL 100 243201 (2008)

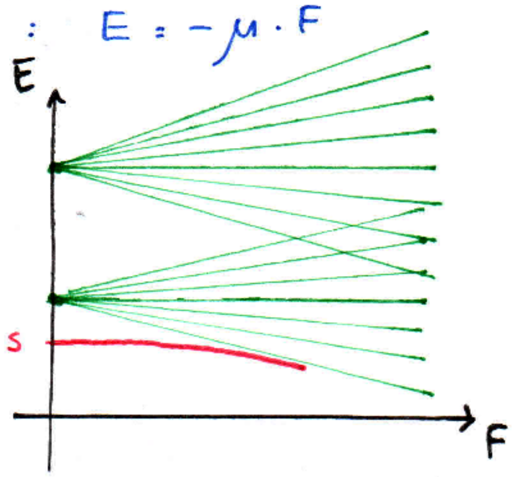
$50s + 50s$ (vdW) $C_6 = 16\,\mathrm{GHz}\,\mu m^6$

M. Saffman and T. Walker
'Quantum information with Rydberg atoms'
Rev. Mod. Phys. 82 2313 (2010)

Permanent electric dipoles

Energy of a dipole in an e-field F is: $E = -\mu \cdot F$

Dipole moment is $\mu(F) = -\frac{dE}{dF}$



Low angular momentum states do not have a permanent dipole moment but applying a small e-field mixes states with different angular momentum.

For the fully stretched state: $\mu_d = \frac{-3}{2} n^*(n^*-1)\, e a_0$

For $n=50$ $\mu_d \simeq 9000\,D$ $(1D = 0.39\, ea_0)$

[LiCs molecules $\mu_{LiCs} = 5.5\,D$]

2.2 Excitation blockade

Typically interactions dominate all other energy scales in the system

$V_{RR} \sim 10-100\,MHz$

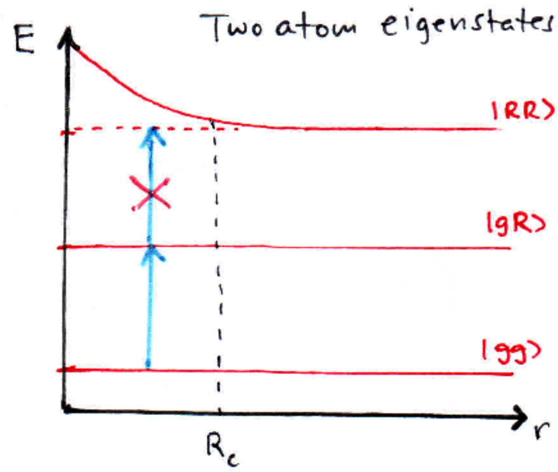
Temperature $kT \sim 20\,kHz/\mu K$ } Frozen regime

trap freq. $\omega \sim 1\,kHz$

The only other relevant parameter is the driving field Rabi freq. $\Omega \sim 1-10\,MHz$

$\therefore$ V_{RR} and Ω determine the properties of the system.

Rydberg blockade

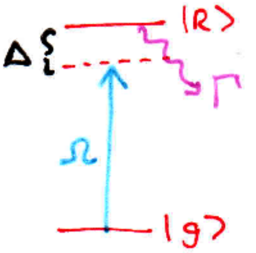


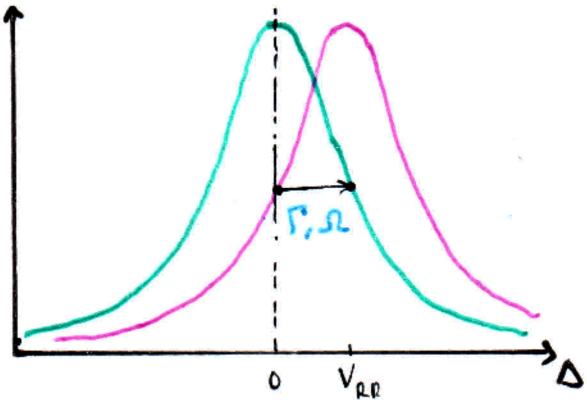
Excitation of one atom strongly shifts the energy of a second nearby atom $\Rightarrow$ Blocked excitation

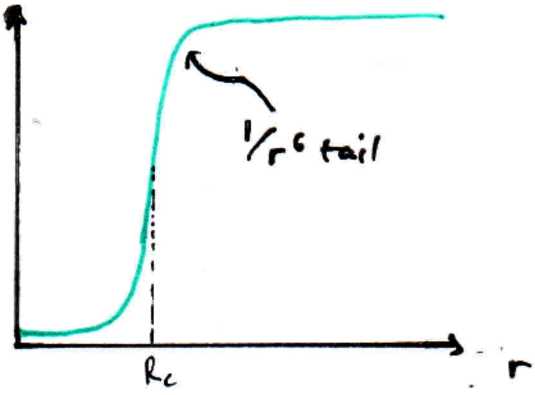
Applications:
- Quantum logic
- Single atom/photons
- Strongly correlated systems.

Blockade Radius (R_c)

The distance between two atoms for which the Rydberg interaction energy equals the excitation bandwidth — laser linewidth Γ
Rabi frequency Ω







Critical radius

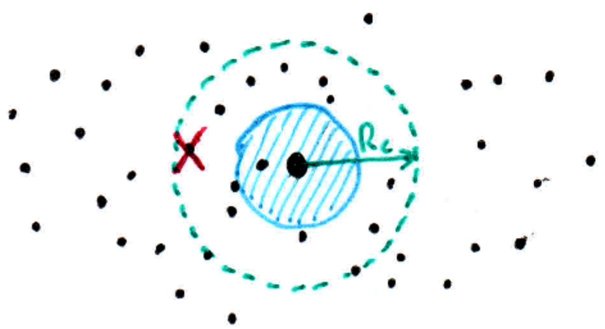
$$V_{RR} = \frac{|C_6|}{r^6} = \sqrt{\Gamma^2 + \Omega^2} \quad \text{so,} \quad R_c = \left(\frac{|C_6|}{\Gamma}\right)^{1/6} \propto (n^*)^{11/6}$$

Typical parameters: $\Gamma/2\pi \sim 1\,\text{MHz}$ $\quad C_6/2\pi \sim 16\,\text{GHz}\,\mu\text{m}^6$ $(n=50)$

$$R_c \approx 5\,\mu\text{m}.$$

Hard sphere picture

Can only excite a single atom within a sphere of radius R_c
$\Rightarrow$ Strong spatial correlations which resemble a gas of hard core particles



rate equation: (for Rydberg density) $(n - n_R)$

$$\frac{dn_R}{dt} = A\,(1 - n_R V_{bl})\,n_g - B\,n_R$$

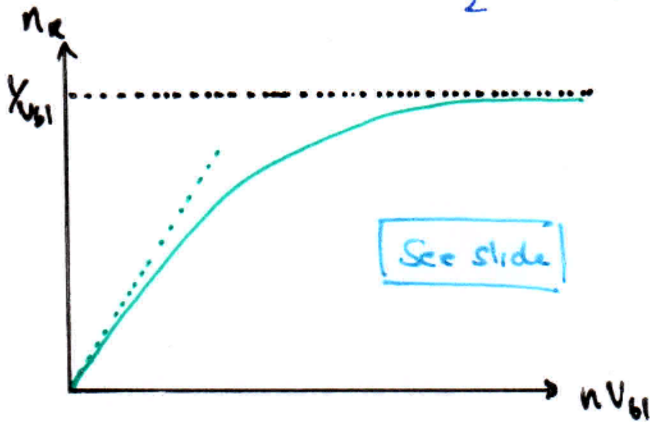
excluded volume term

A : excitation rate.
B : de-excitation rate.
V_{bl} : Blockade volume : $V_{bl} = 4\pi R_c^3/3$

For a two level system $A = B$
Can solve for steady state $(dn_R/dt \to 0)$

$$n_R V_{bl} = 1 + \frac{nV_{bl}}{2} - \sqrt{1 + \left(\frac{nV_{bl}}{2}\right)^2} \simeq 1 - \frac{1}{nV_{bl}} \quad \text{for } n \gg n_R$$

and $\simeq nV_{bl}/2$ for $n \simeq 0$



note : $n_{crit} \approx 1/V_{bl}$

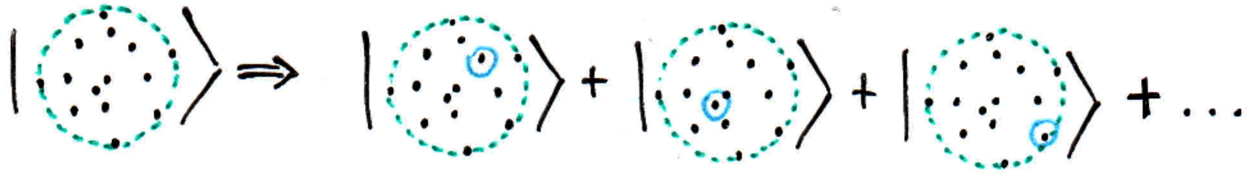
Packing fraction $\eta = n_R \pi R_c^3/6$
$= \dfrac{n_R V_{bl}}{8}$

$\eta_{max} = 0.125$ (max packing fraction for hard spheres $\eta = 0.74$)

Superatoms.

Laser excitation is a coherent process

Can think of the gas as comprised of a collection of superatoms
- effective two-level systems, each involving many atoms.



$|g_N\rangle = |ggg..g_N\rangle$ $\qquad$ $|R_N\rangle = \frac{1}{\sqrt{N}}\sum_i ggg\ldots R_i \ldots g_N\rangle$

A convenient basis is formed by the many-body states

$$|R_N^{(i)}\rangle = |g\rangle_1|g\rangle_2|g\rangle_3\ldots|R\rangle_i\ldots|g\rangle_N$$

The ground state is

$$|g_N\rangle = |g\rangle_1|g\rangle_2|g\rangle_3\ldots|g\rangle_N$$

The laser field couples $|g_N\rangle$ to the <u>collective</u> state $|R_N\rangle$

$$|R_N\rangle = \frac{1}{\sqrt{N}}\sum_i |R_N^{(i)}\rangle$$

Normalization

The coupling operator $\hat{\Omega} = \Omega\sum_{j=1}^{N}\left(|R\rangle_j\langle g|_j + |g\rangle_j\langle R|_j\right)\otimes \mathbb{1}_{j+1}\ldots$

each atom is coupled from $|g\rangle$ to $|R\rangle$ with Rabi frequency Ω

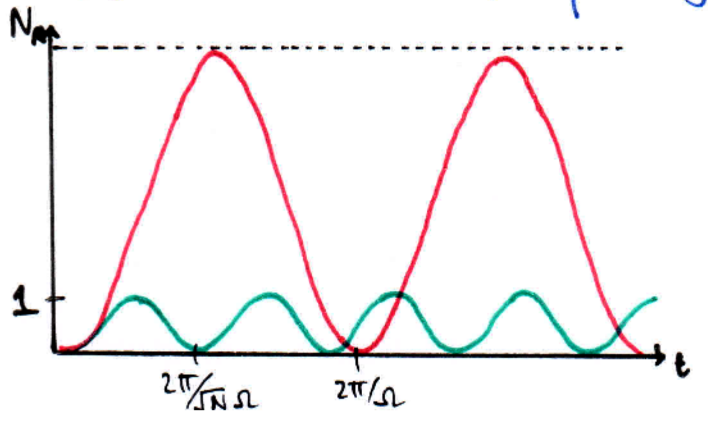
What is the coupling to the collective state $|R_N\rangle$?

$$\langle R_N|\hat{\Omega}|g_N\rangle = \frac{1}{\sqrt{N}}\Omega\sum_i\sum_j \underbrace{\langle R_N^{(i)}|R_j\rangle}_{\delta_{ij}}\langle g_j|g_N\rangle$$

$$= \frac{1}{\sqrt{N}}\Omega\sum_i 1 = \boxed{\sqrt{N}\,\Omega}$$

$\boxed{\text{see slide}}$

The effective Rabi frequency is enhanced by a factor $\sqrt{N}$



Question: how does the collective enhancement affect the blockade volume?

Universal behavior – Hallmark of strongly correlated systems.

H. Weimer, R. Löw, T. Pfau and H-P. Büchler – PRL 101 250601 (2008)

Hamiltonian

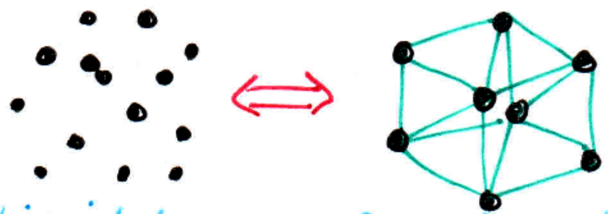
$$\hat{H} = \underbrace{-\Delta \sum_i \hat{\sigma}_{RR}^{(i)}}_{\substack{\text{Laser detuning} \\ \text{Longitudinal field}}} + \underbrace{\frac{\Omega}{2} \sum_i \left(\hat{\sigma}_{gR}^{(i)} + \hat{\sigma}_{Rg}^{(i)} \right)}_{\substack{\text{excitation / coupling} \\ \text{transverse field}}} + \underbrace{\sum_{i<j} V_{ij}\, \sigma_{RR}^{(i)} \sigma_{RR}^{(j)}}_{\substack{\text{Repulsive vdW interaction} \\ \text{Spin-Spin interaction}}}$$

transition operators $\hat{\sigma}_{\alpha\beta} = |\alpha\rangle\langle\beta|$ where $\alpha, \beta = \{g, R\}$

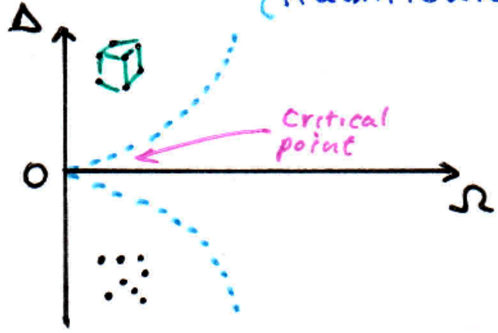
$V_{ij} = -C_6 / |r_i - r_j|^6$ — $r_{i,j}$ can be positions on a lattice or random positions in a gas.

Essentially equivalent to a magnetic spin system – {Quantum Ising Hamiltonian.}

>> Quantum phase transition



Liquid phase ⟺ Crystalline phase



Two regimes

① $\Omega \to 0$ 'Classical regime'

$\Delta < 0$ Ground state is 'paramagnetic' $|ggg \cdots g_N\rangle$

$\Delta > 0$ Some Rydberg excitations present – minimal energy state HCP ordered

② $\Delta = 0$ (finite Ω) "Critical regime"

Diverging correlation length (scale invariant) $R_c \to \infty$

System characterized by a single parameter

$$\beta = \frac{C_6 n^2}{\Omega} \propto \left(n V_{bl}^{(0)} \right)^2$$

($n V_{bl}^{(0)}$: No. of atoms per blockade sphere)

Ratio of interaction energy to driving "Coupling parameter"

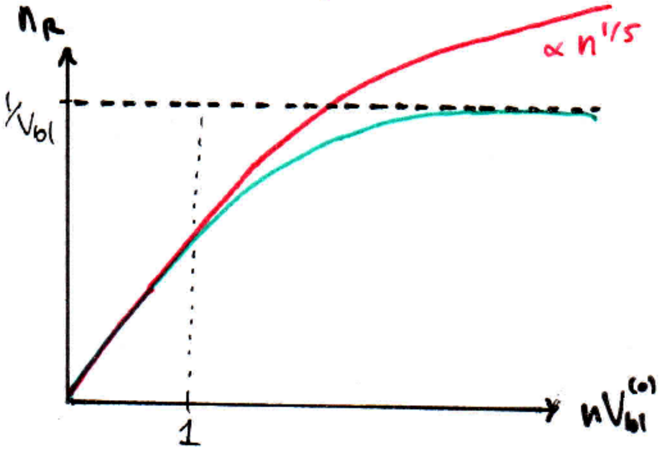
Order parameter – Rydberg fraction

$$f_R = \frac{n_R}{n} \propto \beta^{-\nu} \qquad \nu : \text{universal scaling exponent}$$

Using a mean field theory with $g^{(2)} = \Theta(|r| - a_R)$

find in the limit $\beta \gg 1$ $\nu = 2/5$

for $f_R = \beta^{-2/5} \propto (n V_{bl}^{(0)})^{-4/5}$ Rydberg density doesn't saturate



Hard sphere model:

$$f_R^{(HS)} = \frac{n_R}{n} = \left(\frac{1}{V_{bl}}\right)\frac{1}{n} \propto N_{bl}^{-1}$$

but collective enhancement

$$V_{bl} = (V_{bl}^{(0)})^{4/5} n^{-1/5}$$

$$f_R^{(HS)} \propto (n V_{bl}^{(0)})^{-4/5}$$

Comparison – Experiment vs. theory

$\boxed{\text{R. Löw et al – PRA 80, 033422 (2009)}}$

"Universal scaling in a strongly interacting Rydberg gas"

Expt: $N = 10^5 - 10^7$ Rb-87 atoms in a magnetic trap.

Cigar shaped atomic cloud. $\sigma_r \sim 9\,\mu m$

Rydberg excitation $|R\rangle = 43S_{1/2}$ $R_c \sim 5\,\mu m$

$N_{bl} \sim 100 - 10{,}000$ (crossover btw 3D and 1D)

Must account for inhomogeneous density distribution

$$n(r) = n_0 e^{-r^2/2\sigma^2} \quad \text{(Gaussian cloud shape)}$$

Local density approximation $\beta(r) = \dfrac{C_6 n(r)^2}{\Omega}$

$$f_R^{tot} = \frac{1}{N}\int dr^3 f_R(r) n(r) \propto \beta^{-\nu} / (1-2\nu)^{3/2}$$

Cloud integrated Rydberg fraction

Scaling not affected by trapping potential!!

See slide

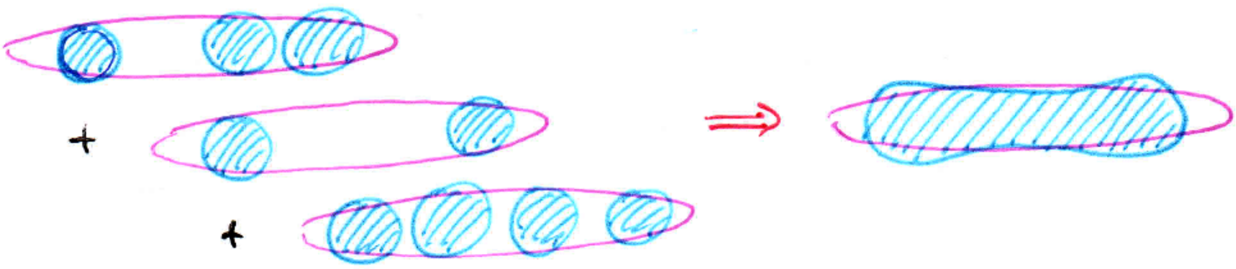
Result: Assuming 3D scaling | 1D scaling
$\nu_{exp} = 0.45 \pm 0.01$ | $\nu_{exp} = 0.16 \pm 0.01$
$\nu_{theor} = 0.4$ | $\nu_{theor} = 0.15$

Rydberg crystals

T. Pohl, E. Demler and M. Lukin, PRL 104, 043002 (2010)

Extension of the single atom blockade to larger systems

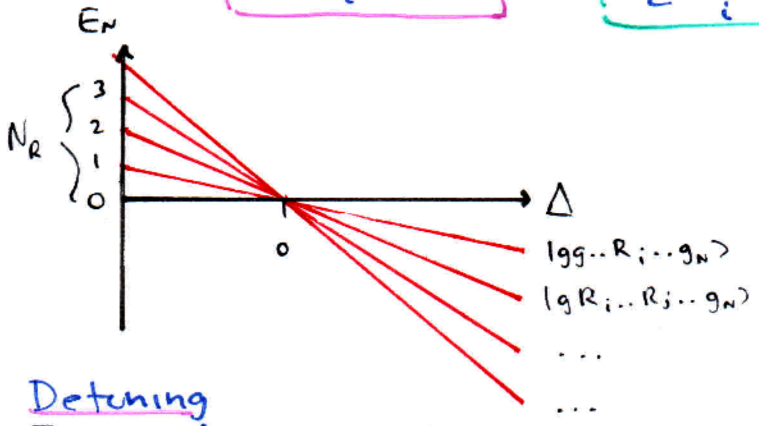
→ Even in the strongly blockaded regime relaxation to steady state following pulsed excitation leads to uncertainty in the number of excited Rydberg atoms.



Single shot distributions — Average density distribution

Another way - chirped excitation

$$\hat{H} = \boxed{-\Delta \sum_i \hat{\sigma}_{RR}^{(i)}} + \boxed{\frac{\Omega}{2} \sum_i \left(\hat{\sigma}_{gR}^{(i)} + \hat{\sigma}_{Rg}^{(i)}\right)} + \boxed{\sum_{i<j} V_{ij}\, \hat{\sigma}_{RR}^{(i)} \hat{\sigma}_{RR}^{(j)}}$$



See slide

Courtesy Rick van Bijnen

Detuning
$E_N = \Delta$ times the number of Rydberg excited atoms.

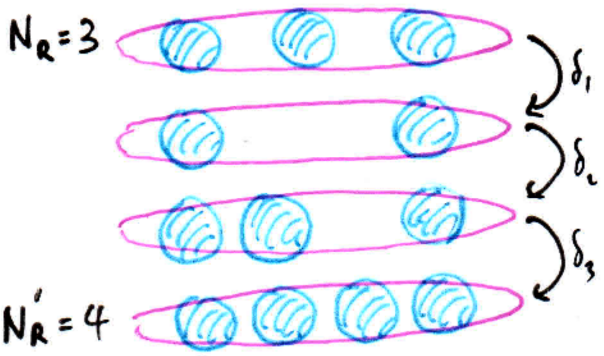
Interactions - shift each line depending on the particular configuration. Lowest energy states correspond to maximum distances between Rydberg atoms (spatial ordering)

Laser coupling - Couples states with different N_R for sufficiently large Ω - level crossings become avoided crossings and the lowest energy state separates from the rest.

1) Start with laser tuned below resonance ($\Delta < 0$)
2) Increase laser intensity (increase Ω)
3) Sweep detuning from below resonance to above resonance
4) Ramp off laser field to approach ground state of the system.

Adiabatic criteria

Evolution must be slow compared to the inverse energy gap between adjacent crystal states



For $N_R \geq 3$ require $2N_R - 3$ steps.

Due to the energy difference for states with different N_R – each step is a virtual transition

» Vanishing multiphoton Rabi coupling for $N_R > 3$: Energy gap $\delta^N_{N+1} \to 0$

Strong coupling regime $\Omega > C_6/a_N^6$ a_N: Rydberg Lattice spacing

Power broadening exceeds the intermediate state detunings such that different crystal states (different N_R) are coupled by sequential single photon couplings

Energy gap $\delta^N_{N+1} \sim \Omega f_R$ Finite gap even for large N_R

Numerical simulations

Cigar (1D) and pancake (2D) shaped Bose-Einstein condensates

$\boxed{\text{See slide}}$

Detection methods

- Statistical means (single atom counting)
- Ion optics (field ion microscope)
- EIT imaging (atom-light mapping)
  - T. Pohl et al, PRL, 104 (2010) Single photon readout
  - G. Günter et al, PRL, 108 (2012) } Interaction enhanced imaging
  - B. Olmos et al, PRA, 84 (2011)
- Lattice imaging P. Schauß et. al. arXiv: 1209.0944

$\boxed{\text{See slide}}$

III. Rydberg dressed quantum gases

Overview: Rydberg dressing theory
Realistic experimental conditions
New quantum phases

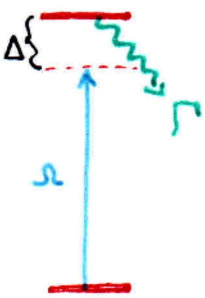
Rydberg dressing. Beyond the 'frozen' regime

Traditionally interactions between ultracold atoms are governed by isotropic - short range interactions (s-wave scattering)

Rydberg dressing: Exploit the properties of Rydberg states by weakly admixing Rydberg character to the gas with laser light

[Santos, Shlyapnikov, Zoller, Lewenstein, PRL, 85 1791 (2000)]

Simple picture



Dressed states $|\Psi\rangle = \alpha|g\rangle + \beta|R\rangle$

weak dressing $\alpha \approx 1$, $\boxed{\beta = \Omega/2\Delta}$

[J. Johnson, S. Rolston PRA, 82, 033412 (2010)]

Essentially the same as what happens in the AC stark shift or light shift

Dressed atom properties

Decay rate: $\Gamma_d \sim |\langle g|\hat{d}|\Psi\rangle|^2$

$\hat{d}$ = dipole operator for spontaneous emission

$$= |\alpha \underbrace{\langle g|\hat{d}|g\rangle}_{=0} + \beta\langle g|\hat{d}|R\rangle|^2 \qquad |\langle g|\hat{d}|R\rangle|^2 = \Gamma$$

$$= \beta^2\,\Gamma$$

Two body interactions: $E_{int} = {}_1\langle\Psi|\,{}_2\langle\Psi|\,\hat{V}_{RR}\,|\Psi\rangle_2\,|\Psi\rangle_1$

$$= \beta^4\,{}_1\langle R|\,{}_2\langle R|\,\hat{V}_{RR}\,|R\rangle_2\,|R\rangle_1 \qquad \hat{V}_{RR} = V_{RR}|RR\rangle\langle RR|$$

$$= \beta^4\,V_{RR}$$

For $\beta \sim 10^{-3}$: Long lifetimes (100's of milliseconds) + New types of interactions (dipole moments of several Debye)

Engineered interactions

- Dipolar quantum gases
- Strongly correlated matter
- New quantum phases
- Soft core interactions
- Quantum magnetism
- Coupled bilayer systems.

Two-atom eigenstates

Separate internal and external degrees of freedom

two atom Hamiltonian

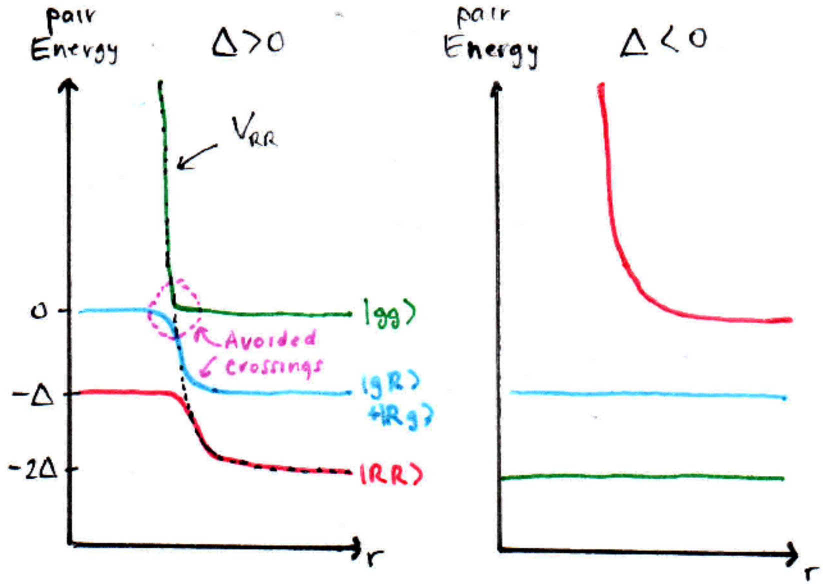
collectively enhanced coupling

$$\hat{H} = \begin{pmatrix} 0 & \Omega/\sqrt{2} & 0 \\ \Omega/\sqrt{2} & -\Delta & \Omega/\sqrt{2} \\ 0 & \Omega/\sqrt{2} & -2\Delta + \boxed{V_{RR}} \end{pmatrix} \qquad |\psi\rangle = \left\{ |gg\rangle, \frac{1}{\sqrt{2}} |gR\rangle + |Rg\rangle, |RR\rangle \right\}$$

Can solve for new dressed state eigenenergies by diagonalizing $\hat{H}$

See slide

Dressed potentials for $\Delta > 0$ and $\Delta < 0$



Effective soft core interaction

The Rydberg blockade has a significant effect on the interaction potential.

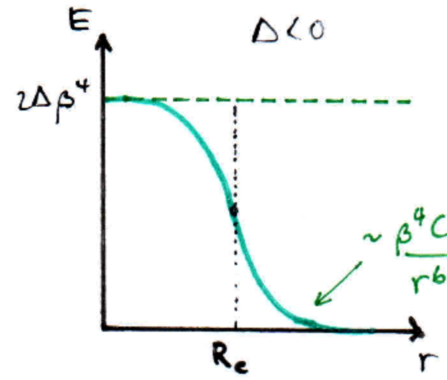
Rydberg-Rydberg interactions lead to an effective detuning of $|RR\rangle$ at short distances which changes β

For $\Delta < 0$: Gives rise to a softened potential

Approximate shape : $\quad V_d \approx \dfrac{\beta^4 C_6}{r^6 + R_c'^6}$

N. Henkel, R. Nath and T. Pohl PRL 104 195302 (2010)

with $R_c' = \left(\dfrac{C_6}{2\Delta}\right)^{1/6}$



For $r \gg R_c \qquad V_d \sim \beta^4 V_{RR}$

For $r \ll R_c \qquad V_d = \Omega^4 / 8\Delta^3$ Independent of interaction strength.

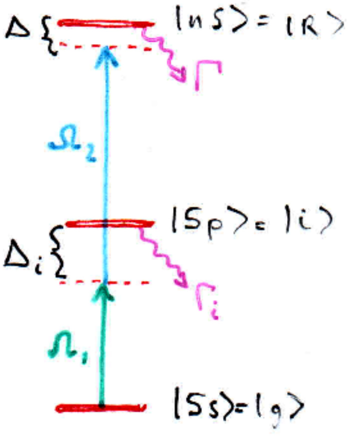
$$= 2\Delta\beta^4$$

Completely new type of interatomic potential !

Full control over all parameters by tuning laser fields

Realistic conditions
(experimental constraints)

Typically Rydberg states are coupled via an (off-resonant) intermediate state



Ladder config.

Short-lived intermediate state $|i\rangle$ decays with rate $\Gamma_i \gg \Gamma$ For Rb-87 $1/\Gamma_i = 26\,\text{ns}$ ☹

For $\Delta_i \gg \Gamma_i$ can "adiabatically eliminate" the intermediate state. [E. Brion, L.H. Pedersen, K. Mølmer J. Phys. A: Math, Theor. 40 1033 (2007)]

Two photon Rabi frequency $\Omega = \dfrac{\Omega_1 \Omega_2}{2\Delta_i}$

Intermediate state population $P_i \simeq \dfrac{\Omega_1^2}{4\Delta_i^2} = \dfrac{\Omega^2}{\Omega_2^2}$

To ensure dressed-state lifetime is not limited by $|i\rangle$

$$\boxed{\Gamma_i P_i \ll \beta^2 \Gamma}$$

$$\therefore \quad \Delta_i^2 \gg \frac{\Omega_1^2}{4} \frac{\Gamma_i}{\Gamma} \frac{1}{\beta^2} \quad \text{and} \quad \boxed{\Delta^2 \ll \frac{1}{4} \frac{\Gamma}{\Gamma_i} \Omega_2^2}$$

Constraint is totally independent of first step excitation laser and Rydberg admixture β^2

Require very high coupling laser intensities ($\propto \Omega_2^2$) and small Rydberg state (two-photon) detunings Δ

Estimated parameters:

$\beta^2 \sim 10^{-3}$ choose Ω_1/Δ_i

$n = 45$ $\Gamma/2\pi \sim 2\,\text{kHz}$ ($\Gamma_d/2\pi \sim 2\,\text{Hz}$ lifetime $\sim 80\,\text{ms}$)

$\Delta/2\pi \sim 1\,\text{MHz}$ Experimentally reasonable - 500x larger than Rydberg linewidth

$\Omega_2/2\pi \gtrsim 110\,\text{MHz}$.   Using transition matrix elements $\mu_s = 4.508\, n^{-3/2} \sqrt{1/3}$
Intensity $\sim 5\,\text{MW/cm}^2$
Power $\sim 8\,\text{W}$ in a waist of $10\,\mu\text{m}$!

Rydberg induced Solitons

The large value of $R_c \approx 5\mu m$ means interactions can no longer be considered point like

Nonlocal Gross-Pitaevskii equation

$$i\hbar \frac{\partial \psi}{\partial t} = \Big[-\frac{\hbar^2 \nabla^2}{2M} + \underbrace{g|\psi|^2}_{\text{repulsive s-wave interactions}} + \int dr' \underbrace{V_d(r-r')}_{\text{soft core potential}} |\psi(r')|^2 \Big] \psi$$

$\boxed{\text{F. Maucher et al, PRL } \underline{106} \text{ 170401 (2011)}}$

- Stable 3D bright solitons in a BEC (self trapping)

Soliton: Shape preserving localized solution to a nonlinear wave equation – Nonlinearity compensates dispersion

Off resonant dressing to Rydberg D-states
> 'Bath tub' potential

$n > 59$ purely attractive interactions

Condensate dynamics following switch off of external confinement

$\boxed{\text{See slide}}$

Strongly correlated quantum gases.

$\boxed{\text{G. Pupillo et al. PRL } \underline{104} \text{ 223002 (2010)}}$

Dressed atoms with dipole-dipole interactions
$\Delta > 0$

dipole moment $\sim$ k Debye regime

2D confinement. Suppression of the attractive part of the dipole-dipole interaction

Monte-Carlo Simulation
+ molecular dynamics simulations

$$\tau = \left(\frac{m\tilde{D}}{\hbar^2 l} \right)^{2/5}$$

effective mass characterizing the strength of interactions.

$$l = \sqrt{\hbar / m\omega}$$

Four different quantum phases:

(i) Superfluid regime
(ii) Supersolid – Simultaneous appearance of solid and superfluid order
(iii) Ring crystal – independent rotational degree of freedom
(iv) Classical crystal. – fixed relative positions.

$\boxed{\text{See slide}}$